Business Statistic

Week 11 Two-Sample Tests

Learning Objectives

The means of two independent populations

The means of two related populations

In this chapter, you learn how to use hypothesis testing for comparing the difference between:

The proportions of two independent populations

The variances of two independent populations by testing the ratio of the two variances

Two-Sample Tests



Difference Between Two Means



Goal: Test hypothesis or form a confidence interval for the difference between two population means, $\mu_1 - \mu_2$

The point estimate for the difference is $\overline{X}_1 - \overline{X}_2$

Difference Between Two Means: Independent Samples



Hypothesis Tests for Two Population Means

Two Population Means, Independent Samples

| Lower-tail test: | Upper-tail test: | Two-tail test: |
|---|---|--|
| $H_0: \mu_1 \ge \mu_2$ $H_1: \mu_1 < \mu_2$ | $H_0: \mu_1 \le \mu_2$ $H_1: \mu_1 > \mu_2$ | $H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$ |
| i.e., | i.e. <i>,</i> | i.e., |
| $ \begin{aligned} &H_0: \mu_1 - \mu_2 \geq 0 \\ &H_1: \mu_1 - \mu_2 < 0 \end{aligned} $ | $ \begin{aligned} &H_0: \mu_1 - \mu_2 \leq 0 \\ &H_1: \mu_1 - \mu_2 > 0 \end{aligned} $ | $H_0: \mu_1 - \mu_2 = 0 H_1: \mu_1 - \mu_2 \neq 0$ |
| | | |

Hypothesis tests for $\mu_1 - \mu_2$



Hypothesis tests for $\mu_1 - \mu_2$ with σ_1 and σ_2 unknown and assumed equal



Assumptions:

- Samples are randomly and independently drawn
- Populations are normally distributed or both sample sizes are at least 30
- Population variances are unknown but assumed equal

Hypothesis tests for $\mu_1 - \mu_2$ with σ_1 and σ_2 unknown and assumed equal (continued)



• The pooled variance is:

$$S_{p}^{2} = \frac{(n_{1} - 1)S_{1}^{2} + (n_{2} - 1)S_{2}^{2}}{(n_{1} - 1) + (n_{2} - 1)}$$

• The test statistic is:



• Where t_{STAT} has **d.f. = (n₁ + n₂ - 2)**

Confidence interval for $\mu_1 - \mu_2$ with σ_1 and σ_2 unknown and assumed equal



The confidence interval for

$$\mu_1 - \mu_2$$
 is:

$$\left(\overline{\mathbf{X}}_{1}-\overline{\mathbf{X}}_{2}\right) \pm t_{\alpha/2} \sqrt{\mathbf{S}_{p}^{2}\left(\frac{1}{\mathbf{n}_{1}}+\frac{1}{\mathbf{n}_{2}}\right)}$$

Where $t_{\alpha/2}$ has **d.f.** = $n_1 + n_2 - 2$

Pooled-Variance t Test Example

You are a financial analyst for a brokerage firm. Is there a difference in dividend yield between stocks listed on the NYSE & NASDAQ? You collect the following data:

| | <u>NYSE</u> | <u>NASDAQ</u> |
|----------------|-------------|---------------|
| Number | 21 | 25 |
| Sample mean | 3.27 | 2.53 |
| Sample std dev | 1.30 | 1.16 |

Assuming both populations are approximately normal with equal variances, is there a difference in mean yield ($\alpha = 0.05$)?



Pooled-Variance t Test Example: Calculating the Test Statistic

(continued)

H0:
$$\mu_1 - \mu_2 = 0$$
 i.e. $(\mu_1 = \mu_2)$
H1: $\mu_1 - \mu_2 \neq 0$ i.e. $(\mu_1 \neq \mu_2)$

The test statistic is:

 $(n_1 - 1) + (n_2 - 1)$

$$t_{\text{STAT}} = \frac{\left(\overline{X}_{1} - \overline{X}_{2}\right) - \left(\mu_{1} - \mu_{2}\right)}{\sqrt{S_{p}^{2}\left(\frac{1}{n_{1}} + \frac{1}{n_{2}}\right)}} = \frac{\left(3.27 - 2.53\right) - 0}{\sqrt{1.5021\left(\frac{1}{21} + \frac{1}{25}\right)}} = 2.040$$

$$\overline{S_{p}^{2}} = \frac{(n_{1} - 1)S_{1}^{2} + (n_{2} - 1)S_{2}^{2}}{(n_{1} - 1) + (n_{2} - 1)} = \frac{(21 - 1)1.30^{2} + (25 - 1)1.16^{2}}{(21 - 1) + (25 - 1)} = 1.5021$$

(21-1)+(25-1)

Pooled-Variance t Test Example: Hypothesis Test Solution

$$H_{0}: \mu_{1} - \mu_{2} = 0 \text{ i.e. } (\mu_{1} = \mu_{2}) \\ H_{1}: \mu_{1} - \mu_{2} \neq 0 \text{ i.e. } (\mu_{1} \neq \mu_{2}) \\ \alpha = 0.05 \\ df = 21 + 25 - 2 = 44 \\ Critical Values: t = \pm 2.0154 \\ Test Statistic: \\ t_{STAT} = \frac{3.27 - 2.53}{\sqrt{1.5021} \left(\frac{1}{21} + \frac{1}{25}\right)} = 2.040 \\ Reject H_{0} \text{ at } \alpha = 0.05 \\ Conclusion: \\ There is evidence of a difference in means. \\ Reject H_{0} \text{ at } \alpha = 0.05 \\ Conclusion: \\ There is evidence of a difference in means. \\ Reject H_{0} \text{ at } \alpha = 0.05 \\ Conclusion: \\ Conclusion:$$

Pooled-Variance t Test Example: Confidence Interval for $\mu_1 - \mu_2$

Since we rejected H_0 can we be 95% confident that $\mu_{NYSE} > \mu_{NASDAQ}$?

95% Confidence Interval for μ_{NYSE} - μ_{NASDAQ}

$$\left(\overline{X}_{1}-\overline{X}_{2}\right) \pm t_{\alpha/2} \sqrt{S_{p}^{2}\left(\frac{1}{n_{1}}+\frac{1}{n_{2}}\right)} = 0.74 \pm 2.0154 \times 0.3628 = (0.009, 1.471)$$

Since 0 is less than the entire interval, we can be 95% confident that $\mu_{\text{NYSE}} > \mu_{\text{NASDAQ}}$

Hypothesis tests for $\mu_1 - \mu_2$ with σ_1 and σ_2 unknown, not assumed equal



Assumptions:

- Samples are randomly and independently drawn
- Populations are normally distributed or both sample sizes are at least 30
- Population variances are unknown and cannot be assumed to be equal

Hypothesis tests for $\mu_1 - \mu_2$ with σ_1 and σ_2 unknown and not assumed equal (continued)



EXERCISE

10.8 (cont'd)

A recent study found that children who watched a cartoon with food advertising ate, on average, 28.5 grams of Goldfish crackers as compared to an average of 19.7 grams of Goldfish crackers for children who watched a cartoon without food advertising. Although there were 118 children in the study, neither the sample size in each group nor the sample standard deviations were reported.

Suppose that there were 59 children in each group, and the sample standard deviation for those children who watched the food ad was 8.6 grams and the sample standard deviation for those children who did not watch the food ad was 7.9 grams. Assuming that the population variances are equal and $\alpha = 0.05$ is there evidence that the mean amount of Goldfish crackers eaten was significantly higher for the children who watched food ads?

10.10 (cont'd)

The Computer Anxiety Rating Scale (CARS) measures an individual's level of computer anxiety, on a scale from 20 (no anxiety) to 100 (highest level of anxiety). Researchers at Miami University administered CARS to 172 business students. One of the objectives of the study was to determine whether there is a difference in the level of computer anxiety experienced by female and male business students. They found the following:

| | Males | Females |
|---|-------|---------|
| Х | 40,26 | 36,85 |
| S | 13,35 | 9,42 |
| n | 100 | 72 |

- a. At the 0.05 level of significance, is there evidence of a difference in the mean computer anxiety experienced by female and male business students?
- b. What assumptions do you have to make about the two populations in order to justify the use of the t test?

A bank with a branch located in a commercial district of a city has the business objective of developing an improved process for serving customers during the noon-to- 1 P.M. lunch period. Management decides to first study the waiting time in the current process. The waiting time is defined as the time that elapses from when the customer enters the line until he or she reaches the teller window. Data are collected from a random sample of 15 customers, and the results (in minutes) are as follows:

| Waiting Time (Bank 1) | | |
|-----------------------|------|--|
| Mean | 4.29 | |
| Standard Deviation | 1.64 | |

Suppose that another branch, located in a residential area, is also concerned with improving the process of serving customers in the noon-to-1 P.M. lunch period. Data are collected from a random sample of 15 customers, and the results are as follows:

| Waiting Time (Bank 2) | | |
|-----------------------|------|--|
| Mean | 7.11 | |
| Standard Deviation | 2.08 | |

Assuming that the population variances from both banks are unequal, is there evidence of a difference in the mean waiting time between the two branches? (Use $\alpha = 0.05$)

10.16 (cont'd)

Do young children use cell phones? Apparently so, according to a recent study, which stated that cell phone users under 12 years of age averaged 137 calls per month as compared to 231 calls per month for cell phone users 13 to 17 years of age. No sample sizes were reported. Suppose that the results were based on samples of 50 cell phone users in each group and that the sample standard deviation for cell phone users under 12 years of age was 51.7 calls per month and the sample standard deviation for cell phone users 13 to 17 years of age was 67.6 calls per month.

- a. Assuming that the variances in the populations of cellphone users are equal, is there evidence of a difference in the mean cell phone usage between cell phone users under 12 years of age and cell phone users 13 to 17 years of age? (Use a 0.05 level of significance.)
- b. In addition to equal variances, what other assumption is necessary in (a)?

Two-Sample Tests



Related Populations The Paired Difference Test



Tests Means of 2 Related Populations

- Paired or matched samples
- Repeated measures (before/after)
- Use difference between paired values:

$$D_i = X_{1i} - X_{2i}$$

- Eliminates Variation Among Subjects
- Assumptions:
 - Both Populations Are Normally Distributed
 - Or, if not Normal, use large samples



The point estimate for the paired Difference population mean μ_D is \overline{D} :



$$\mathbf{S}_{\mathrm{D}} = \sqrt{\frac{\sum_{i=1}^{n} (\mathbf{D}_{i} - \overline{\mathbf{D}})^{2}}{n-1}}$$

The sample standard deviation is S_D

n is the number of pairs in the paired sample

The Paired Difference Test: Finding t_{STAT}



• The test statistic for μ_D is:



Where t_{STAT} has n - 1 d.f.

The Paired Difference Test: Possible Hypotheses



The Paired Difference Confidence Interval

Related samples

The confidence interval for μ_{D} is

$$\overline{D} \pm t_{\alpha/2} \frac{S_{\rm D}}{\sqrt{\rm n}}$$

where
$$S_{D} = \sqrt{\frac{\sum_{i=1}^{n} (D_{i} - \overline{D})^{2}}{n-1}}$$

Paired Difference Test: Example

Assume you send your salespeople to a "customer service" training workshop. Has the training made a difference in the number of complaints? You collect the following data:

| <u>Salesperson</u> | <u>Number of</u> <u>Before (1)</u> | <u>Complaints</u> : <u>After (2)</u> | (2) - (1) <u>Difference, D_i</u> | $\overline{D} = \frac{\sum D_i}{n}$ |
|--------------------|---------------------------------------|---|---|-------------------------------------|
| C.B. | 6 | 4 | - 2 | = -4.2 |
| T.F. | 20 | 6 | -14 | |
| M.H. | 3 | 2 | - 1 | |
| R.K. | 0 | 0 | 0 | $\sum (D_i - \overline{D})^2$ |
| M.O. | 4 | 0 | <u>- 4</u> | $S_{D} = \sqrt{\frac{2}{n-1}}$ |
| | | | -21 | |
| | | | | 5.67 |

Paired Difference Test: Solution

 Has the training made a difference in the number of complaints (at the 0.01 level)?

Reject Reject $\begin{array}{l} {\rm H_{0}:} \ {\rm \mu_{D}=0} \\ {\rm H_{1}:} \ {\rm \mu_{D}\neq0} \end{array}$ α/2 $\alpha/2$ $\alpha = .01$ $\overline{D} = -4.2$ - 4.604 4.604 - 1.66 $t_{0.005} = \pm 4.604$ Decision: Do not reject H_0 (t_{stat} is not in the reject region) d.f. = n - 1 = 4**Test Statistic: Conclusion: There is insufficient evidence** there is significant change in the number $=\frac{D-\mu_{\rm D}}{S_{\rm D}/\sqrt{n}}=\frac{-4.2-0}{5.67/\sqrt{5}}$ of complaints. 1.66 ι_{stat}

EXERCISE

10.20 (cont'd)

Nine experts rated two brands of Colombian coffee in a taste-testing experiment. A rating on a 7point scale (1 = extremely)unpleasing, 7 = extremely pleasing) is given for each of four characteristics: taste, aroma, richness, and acidity. The following data display the ratings accumulated over all four characteristics.

| | Brand | | |
|---------|-------|----|--|
| Experts | Α | В | |
| C.C. | 24 | 26 | |
| S.E. | 27 | 27 | |
| E.G. | 19 | 22 | |
| B.L. | 24 | 27 | |
| C.M. | 22 | 25 | |
| C.N. | 26 | 27 | |
| G.N. | 27 | 26 | |
| R.M. | 25 | 27 | |
| P.V. | 22 | 23 | |

- a. At the 0.05 level of significance, is there evidence of a difference in the mean ratings between the two brands?
- b. What assumption is necessary about the population distribution in order to perform this test?

10.24 (cont'd)

Multiple myeloma, or blood plasma cancer, is characterized by increased blood vessel formulation (angiogenesis) in the bone marrow that is a predictive factor in survival. One treatment approach used for multiple myeloma is stem cell transplantation with the patient's own stem cells. The following data represent the bone marrow micro vessel density for patients who had a complete response to the stem cell transplant (as measured by blood and urine tests).

10.24 (cont'd)

| Patient | Before | After |
|---------|--------|-------|
| 1 | 158 | 284 |
| 2 | 189 | 214 |
| 3 | 202 | 101 |
| 4 | 353 | 227 |
| 5 | 416 | 290 |
| 6 | 426 | 176 |
| 7 | 441 | 290 |

The measurements were taken immediately prior to the stem cell transplant and at the time the complete response was determined.

At the 0.05 level of significance, is there evidence that the mean bone marrow micro vessel density is higher before the stem cell transplant than after the stem cell transplant?

Two-Sample Tests



Two Population Proportions

Population proportions Goal: test a hypothesis or form a confidence interval for the difference between two population proportions,

$$\pi_1 - \pi_2$$

The point estimate for the difference is

$$p_{1} - p_{2}$$

Two Population Proportions

Population proportions In the null hypothesis we assume the null hypothesis is true, so we assume $\pi_1 = \pi_2$ and pool the two sample estimates

The pooled estimate for the overall proportion is:

$$\overline{p} = \frac{\mathbf{X}_1 + \mathbf{X}_2}{\mathbf{n}_1 + \mathbf{n}_2}$$

where X_1 and X_2 are the number of items of interest in samples 1 and 2

Two Population Proportions

(continued)

The test statistic for

 $\pi_1 - \pi_2$ is a Z statistic:

Population proportions

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\overline{p}(1 - \overline{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where
$$\overline{p} = \frac{X_1 + X_2}{n_1 + n_2}$$
, $p_1 = \frac{X_1}{n_1}$, $p_2 = \frac{X_2}{n_2}$

Hypothesis Tests for Two Population Proportions



Hypothesis Tests for Two Population Proportions

(continued)

Population proportions Upper-tail test: Lower-tail test: Two-tail test: $H_0: \pi_1 - \pi_2 \ge 0$ $H_0: \pi_1 - \pi_2 \le 0$ $H_0: \pi_1 - \pi_2 = 0$ $H_1: \pi_1 - \pi_2 < 0$ $H_1: \pi_1 - \pi_2 > 0$ $H_1: \pi_1 - \pi_2 \neq 0$ **C**/2 **C**/2 α α $-Z_{\alpha/2}$ Z_{α} $Z_{\alpha/2}$ Z_{α} Reject H_0 if $Z_{STAT} > Z_{\alpha}$ Reject H₀ if $Z_{STAT} < -Z_{\alpha/2}$ Reject H_0 if $Z_{STAT} < -Z_{\alpha}$ or $Z_{STAT} > Z_{\alpha/2}$

Hypothesis Test Example: Two population Proportions

Is there a significant difference between the proportion of men and the proportion of women who will vote Yes on Proposition A?

- In a random sample, 36 of 72 men and 35 of 50 women indicated they would vote Yes
- Test at the .05 level of significance



Hypothesis Test Example: Two population Proportions

(continued)

• The hypothesis test is:

 $H_0: \pi_1 - \pi_2 = 0$ (the two proportions are equal) $H_1: \pi_1 - \pi_2 \neq 0$ (there is a significant difference between proportions)

• The sample proportions are:

Women:
$$p_2 = 35/50 = 0.70$$

The pooled estimate for the overall proportion is:

$$\overline{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{36 + 35}{72 + 50} = \frac{71}{122} = 0.582$$

Hypothesis Test Example: Two population Proportions

(continued)



yes between men and women.

Critical Values = ± 1.96 For $\alpha = .05$

Confidence Interval for Two Population Proportions

Population proportions The confidence interval for $\pi_1 - \pi_2$ is:

$$(p_1 - p_2) \pm Z_{\alpha/2} \sqrt{\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}}$$

Two-Sample Tests



Testing for the Ratio Of Two Population Variances



 $n_2 - 1$ = denominator degrees of freedom

The F Distribution

- The F critical value is found from the F table
- There are two degrees of freedom required: numerator and denominator
- The larger sample variance is always the numerator

• When
$$F_{STAT} = \frac{S_1^2}{S_2^2} df_1 = n_1 - 1; df_2 = n_2 - 1$$

- In the F table,
 - numerator degrees of freedom determine the column
 - denominator degrees of freedom determine the row

Finding the Rejection Region

$$H_{0}: \sigma_{1}^{2} = \sigma_{2}^{2}$$

$$H_{1}: \sigma_{1}^{2} \neq \sigma_{2}^{2}$$

$$H_{1}: \sigma_{1}^{2} \neq \sigma_{2}^{2}$$

$$H_{1}: \sigma_{1}^{2} \geq \sigma_{1}^{2}$$

$$H_{1}: \sigma_{1}^{2} \geq \sigma_{1}^{2}$$

$$H_{1}: \sigma_{1}^{2} \geq \sigma_{1}^{2}$$

$$H_{1}: \sigma_{1}^{2} \geq \sigma_{1}^{2}$$

$$H_{1}: \sigma_{1}^{2} \geq \sigma_{1}^{2} \geq \sigma_{1}^{2}$$

$$H_{1}: \sigma_{1}^{2} \geq \sigma_{1}^{2} \geq \sigma_{1}^{2} \geq \sigma_{1}^{2} \geq \sigma_{1}^{2} \geq \sigma_{1}^{2} \geq$$

F Test: An Example

You are a financial analyst for a brokerage firm. You want to compare dividend yields between stocks listed on the NYSE & NASDAQ. You collect the following data:

| | <u>NYSE</u> | <u>NASDAQ</u> |
|---------|-------------|---------------|
| Number | 21 | 25 |
| Mean | 3.27 | 2.53 |
| Std dev | 1.30 | 1.16 |

Is there a difference in the variances between the NYSE & NASDAQ at the α = 0.05 level?



F Test: Example Solution

• Form the hypothesis test:

$$H_0: \sigma_1^2 = \sigma_2^2$$
$$H_1: \sigma_1^2 \neq \sigma_2^2$$

(there is no difference between variances)(there is a difference between variances)

- Find the F critical value for α = 0.05:
- Numerator d.f. = $n_1 1 = 21 1 = 20$
- Denominator d.f. = n2 1 = 25 1 = 24

•
$$F_{\alpha/2} = F_{.025, 20, 24} = 2.33$$

F Test: Example Solution

(continued)



• Conclusion: There is insufficient evidence of a difference in variances at α = .05

EXERCISE

10.30 (cont'd)

Does it take more effort to be removed from an email list than it used to? A study of 100 large online retailers revealed the following:

| NEED THREE OR MORE CLICKS | | | |
|---------------------------|---------------|----|--|
| | TO BE REMOVED | | |
| YEAR | Yes | Νο | |
| 2009 | 39 | 61 | |
| 2008 | 7 | 93 | |

- Set up the null and alternative hypotheses to try to determine whether it takes more effort to be removed from an email list than it used to.
- b. Conduct the hypothesis test defined in (a), using the 0.05 level of significance.
- c. Does the result of your test in (b) make it appropriate to claim that it takes more effort to be removed from an email list than it used to?

How do Americans feel about ads on websites? A survey of 1,000 adult Internet users found that 670 opposed ads on websites. Suppose that a survey of 1,000 Internet users age 12–17 found that 510 opposed ads on websites. At the 0.05 level of significance, is there evidence of a difference between adult Internet users and Internet users age 12–17 in the proportion who oppose ads?

10.46 (cont'd)

The Computer Anxiety Rating Scale (CARS) measures an individual's level of computer anxiety, on a scale from 20 (no anxiety) to 100 (highest level of anxiety). Researchers at Miami University administered CARS to 172 business students. One of the objectives of the study was to determine whether there is a difference between the level of computer anxiety experienced by female students and male students. They found the following:

| | Males | Females |
|---|-------|---------|
| Х | 40.26 | 36.85 |
| S | 13.35 | 9.42 |
| n | 100 | 72 |

- a. At the 0.05 level of significance, is there evidence of a difference in the variability of the computer anxiety experienced by males and females?
- b. What assumption do you need to make about the two populations in order to justify the use of the *F* test?
- c. Based on (a), which t test defined in problem 10.10 should you use to test whether there is a significant difference in mean computer anxiety for female and male students?

THANK YOU