Business Statistic

Week 12

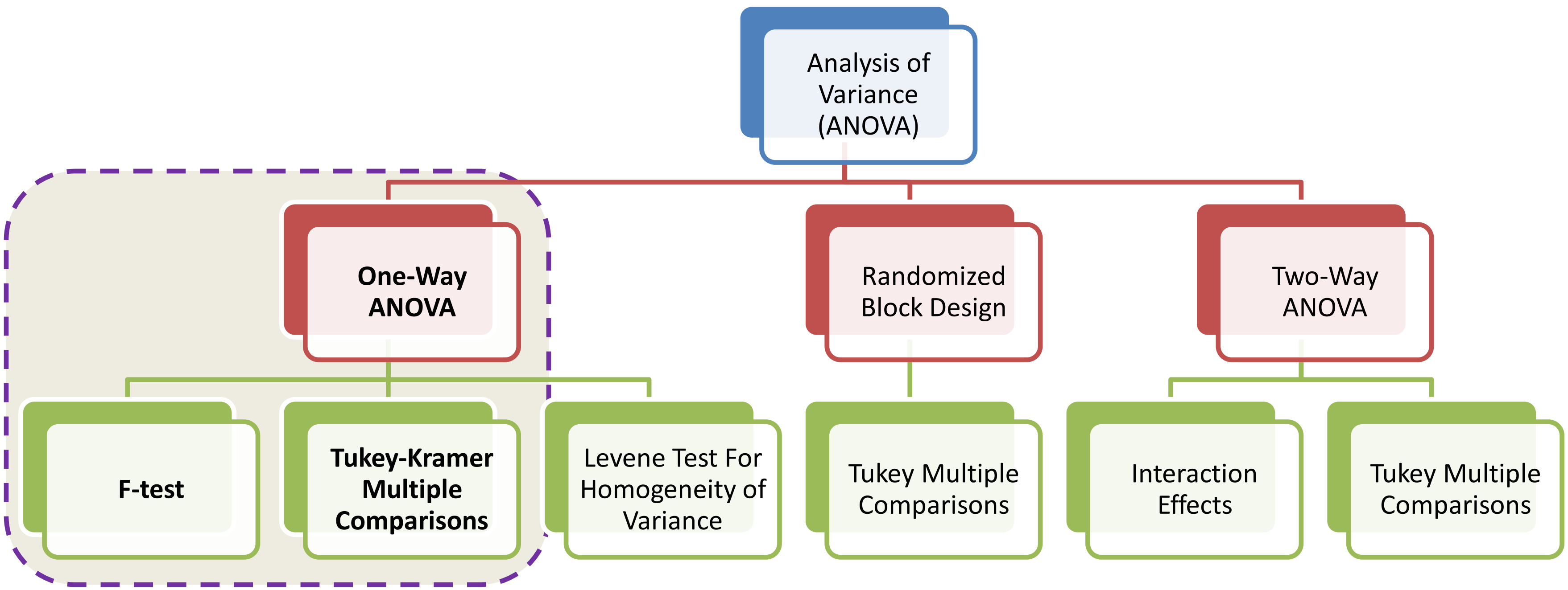
Analysis of Variance

Learning Objectives

This week, you learn:

- How to use one-way analysis of variance to test for differences among the means of several populations (also referred to as "groups" in this chapter)

Chapter Overview



F TEST OF ANOVA

One-Way Analysis of Variance

- Evaluate the difference among the means of three or more groups

Examples: Accident rates for 1st, 2nd, and 3rd shift
Expected mileage for five brands of tires

- Assumptions
  - Populations are normally distributed
  - Populations have equal variances
  - Samples are randomly and independently drawn

Hypotheses of One-Way ANOVA

- $H_0 : \mu_1 = \mu_2 = \mu_3 = \Lambda = \mu_c$
  - All population means are equal
  - i.e., no factor effect (no variation in means among groups)
- $H_1 : \text{Not all of the population means are the same}$
  - At least one population mean is different
  - i.e., there is a factor effect
  - Does not mean that all population means are different (some pairs may be the same)

One-Way ANOVA

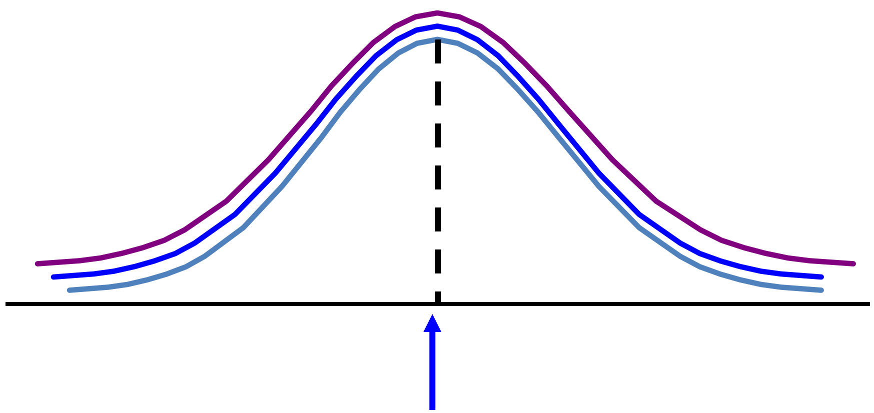
$$H_0 : \mu_1 = \mu_2 = \mu_3 = \Lambda = \mu_c$$

$$H_1 : \text{Not all } \mu_j \text{ are the same}$$

The Null Hypothesis is True

All Means are the same:

(No Factor Effect)



$$\mu_1 = \mu_2 = \mu_3$$

One-Way ANOVA

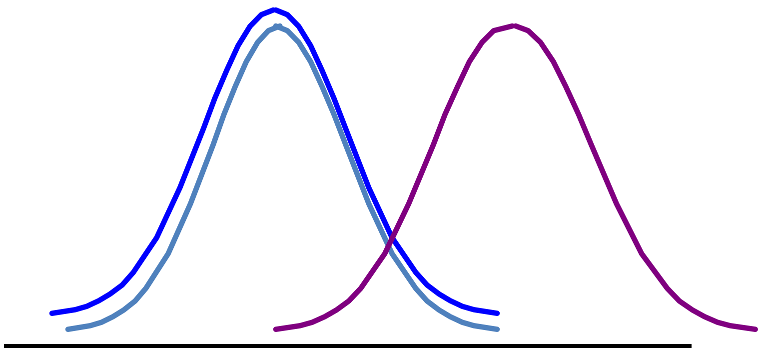
$H_0 : \mu_1 = \mu_2 = \mu_3 = \Lambda = \mu_c$

$H_1 :$ Not all μ_j are the same

The Null Hypothesis is NOT true

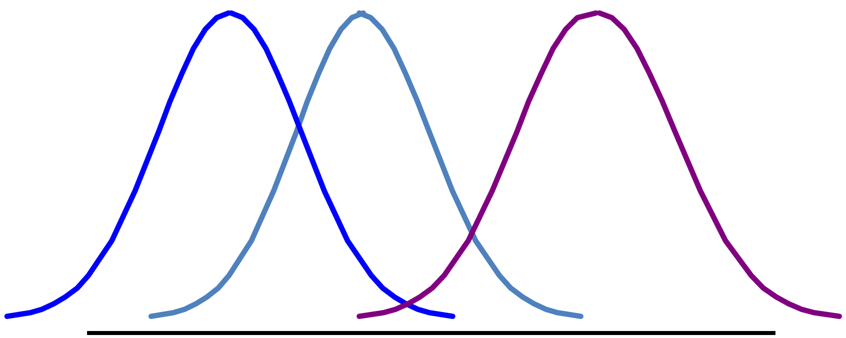
At least one of the means is different

(Factor Effect is present)



$\mu_1 = \mu_2 \neq \mu_3$

or



$\mu_1 \neq \mu_2 \neq \mu_3$

Partitioning the Variation

- Total variation can be split into two parts:

$$SST = SSA + SSW$$

SST = Total Sum of Squares
(Total variation)
SSA = Sum of Squares Among Groups
(Among-group variation)
SSW = Sum of Squares Within Groups
(Within-group variation)

Partitioning the Variation

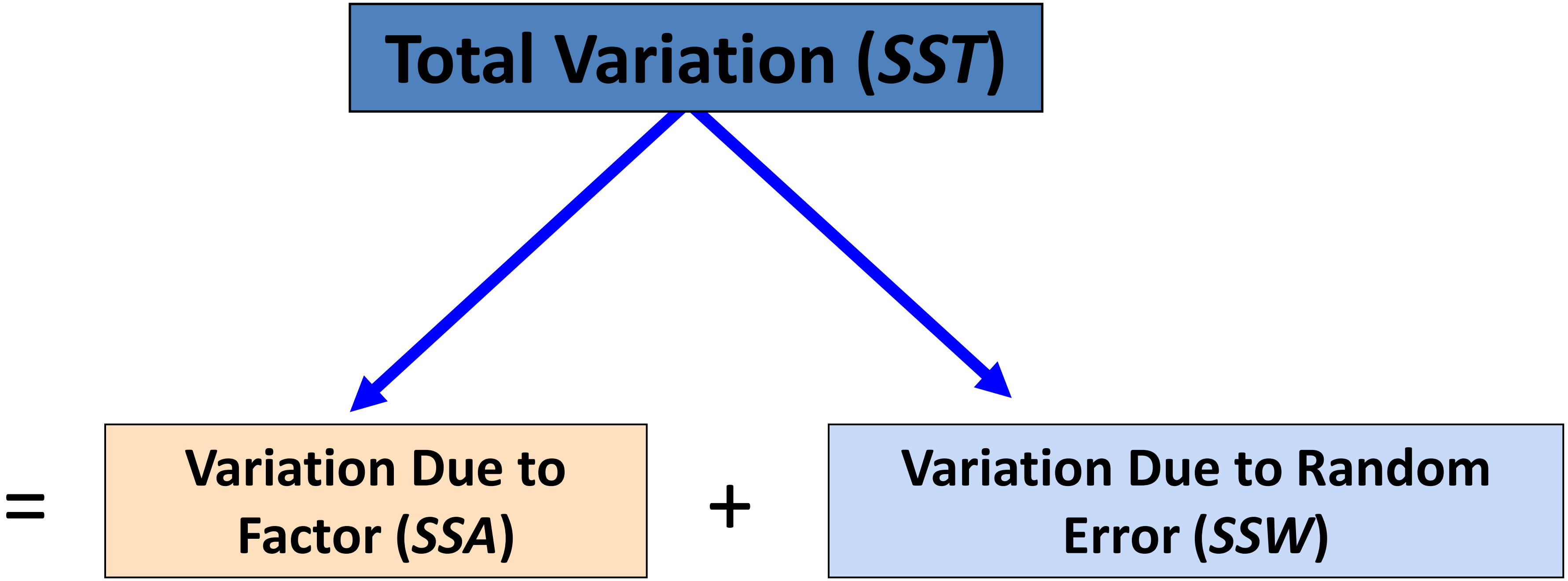
$$SST = SSA + SSW$$

Total Variation = the aggregate variation of the individual data values across the various factor levels (*SST*)

Among-Group Variation = variation among the factor sample means (*SSA*)

Within-Group Variation = variation that exists among the data values within a particular factor level (*SSW*)

Partition of Total Variation



Total Sum of Squares

$$SST = SSA + SSW$$

$$SST = \sum_{j=1}^{c} \sum_{i=1}^{n_j} (X_{ij} - \bar{\bar{X}})^2$$

Where:

SST = Total sum of squares

c = number of groups or levels

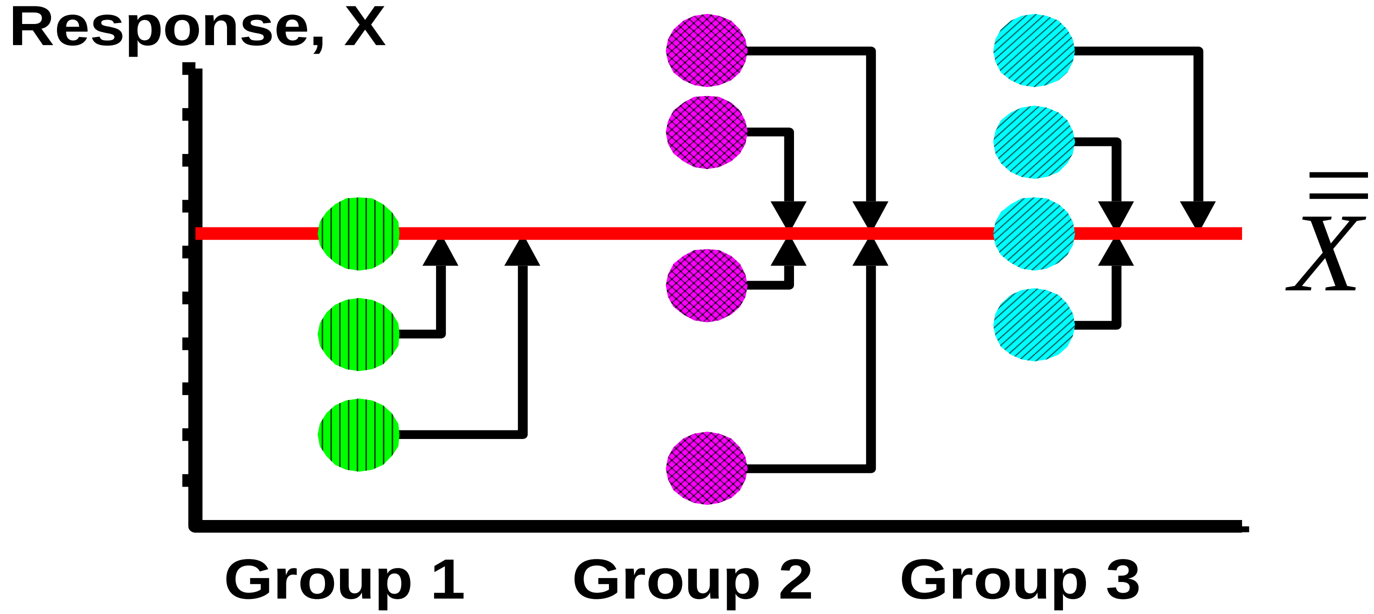
n_j = number of observations in group j

X_{ij} = i^{th} observation from group j

$\bar{\bar{X}}$ = grand mean (mean of all data values)

Total Variation

$$SST = (X_{11} - \bar{\bar{X}})^2 + (X_{12} - \bar{\bar{X}})^2 + \cdots + (X_{cn_c} - \bar{\bar{X}})^2$$



Among-Group Variation

$$SST = \boxed{SSA} + SSW$$

$$SSA = \sum_{j=1}^{c} n_j(\bar{X}_j - \bar{\bar{X}})^2$$

Where:

SSA = Sum of squares among groups

c = number of groups

n_j = sample size from group j

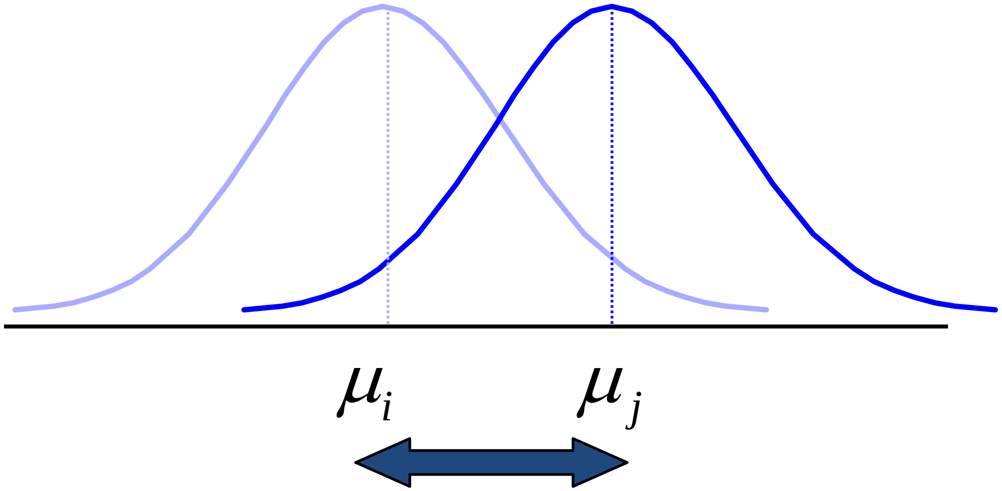
$\bar{X}_j$ = sample mean from group j

$\bar{\bar{X}}$ = grand mean (mean of all data values)

Among-Group Variation

$$SSA = \sum_{j=1}^{c} n_j (\overline{X}_j - \overline{\overline{X}})^2$$

Variation Due to Differences Among Groups

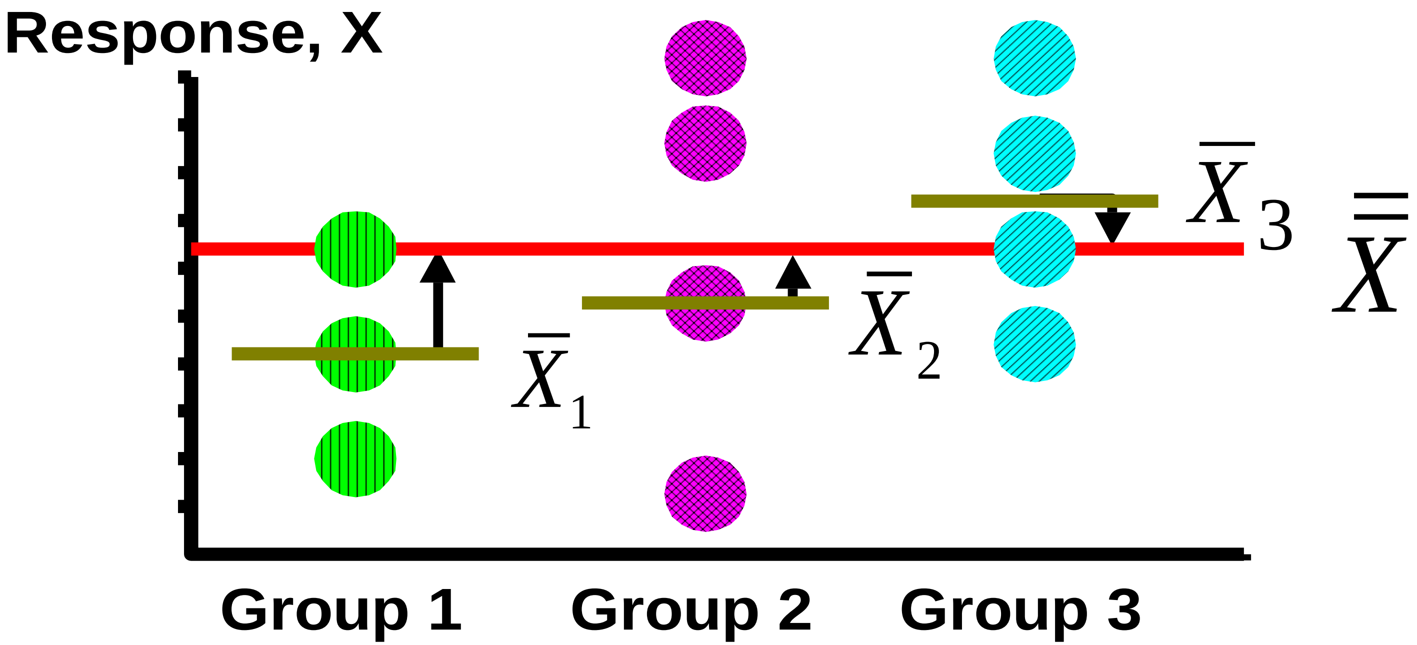


$$MSA = \frac{SSA}{c-1}$$

Mean Square Among = SSA/degrees of freedom

Among-Group Variation

$$SSA = n_1(\overline{X}_1 - \overline{\overline{X}})^2 + n_2(\overline{X}_2 - \overline{\overline{X}})^2 + \cdots + n_c(\overline{X}_c - \overline{\overline{X}})^2$$



Within-Group Variation

$$SST = SSA + SSW$$

$$SSW = \sum_{j=1}^{c} \sum_{i=1}^{n_j} (X_{ij} - \overline{X}_j)^2$$

Where:

SSW = Sum of squares within groups

c = number of groups

n_j = sample size from group j

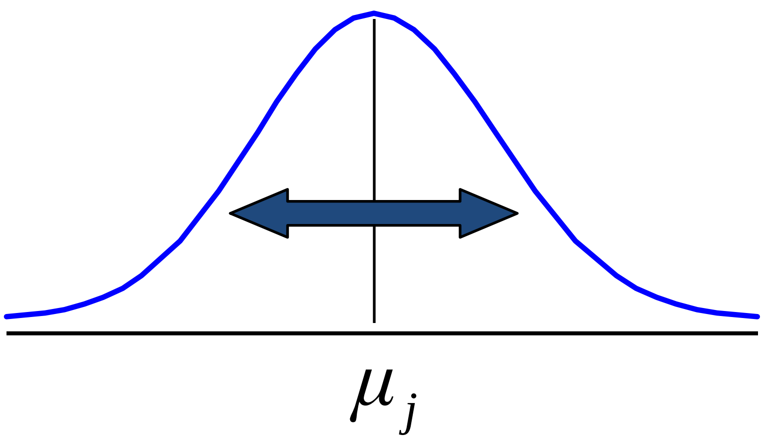
$\overline{X}_j$ = sample mean from group j

X_{ij} = i^{th} observation in group j

Within-Group Variation

$$SSW = \sum_{j=1}^{c} \sum_{i=1}^{n_j} (X_{ij} - \overline{X}_j)^2$$

Summing the variation within each group and then adding over all groups

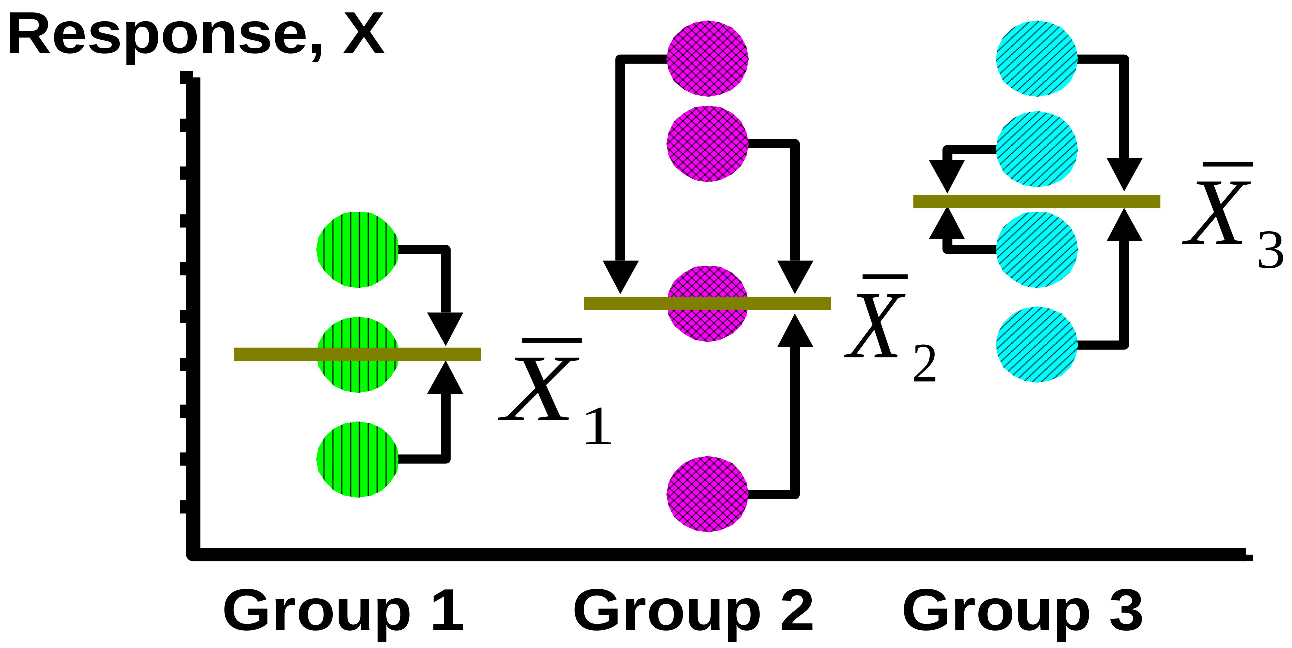


$$MSW = \frac{SSW}{n-c}$$

Mean Square Within = SSW/degrees of freedom

Within-Group Variation

$$SSW = (X_{11} - \overline{X}_1)^2 + (X_{12} - \overline{X}_2)^2 + \cdots + (X_{cn_c} - \overline{X}_c)^2$$



Obtaining the Mean Squares

The Mean Squares are obtained by dividing the various sum of squares by their associated degrees of freedom

$$MSA = \frac{SSA}{c-1}$$

Mean Square Among
(d.f. = c-1)

$$MSW = \frac{SSW}{n-c}$$

Mean Square Within
(d.f. = n-c)

$$MST = \frac{SST}{n-1}$$

Mean Square Total
(d.f. = n-1)

One-Way ANOVA Table

Source of Variation	Degrees of Freedom	Sum Of Squares	Mean Square (Variance)	F
Among Groups	c - 1	*SSA*	$MSA = \frac{SSA}{c - 1}$	$F_{STAT} = \frac{MSA}{MSW}$
Within Groups	n - c	*SSW*	$MSW = \frac{SSW}{n - c}$	
Total	n – 1	*SST*		

c = number of groups

n = sum of the sample sizes from all groups

df = degrees of freedom

One-Way ANOVA F Test Statistic

H_0: $\mu_1 = \mu_2 = \cdots = \mu_c$

H_1: At least two population means are different

- Test statistic

$$F_{STAT} = \frac{MSA}{MSW}$$

MSA is mean squares among groups

MSW is mean squares within groups

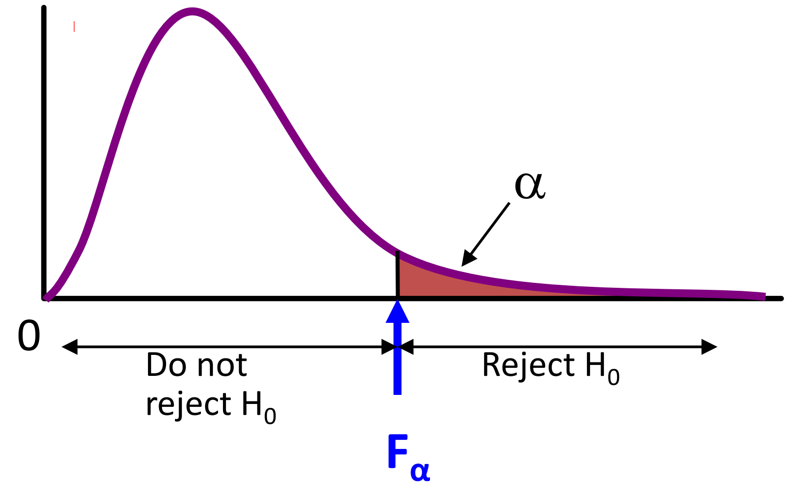
- Degrees of freedom
  - $df_1 = c - 1$ (c = number of groups)
  - $df_2 = n - c$ (n = sum of sample sizes from all populations)

Interpreting One-Way ANOVA F Statistic

- The F statistic is the ratio of the among estimate of variance and the within estimate of variance
  - The ratio must always be positive
  - $df_1 = c - 1$ will typically be small
  - $df_2 = n - c$ will typically be large

Decision Rule:

- Reject H_0 if $F_{STAT} > F_{\alpha}$, otherwise do not reject H_0



One-Way ANOVA F Test Example

You want to see if when three different golf clubs are used, they hit the ball different distances. You randomly select five measurements from trials on an automated driving machine for each club. At the 0.05 significance level, is there a difference in mean distance?

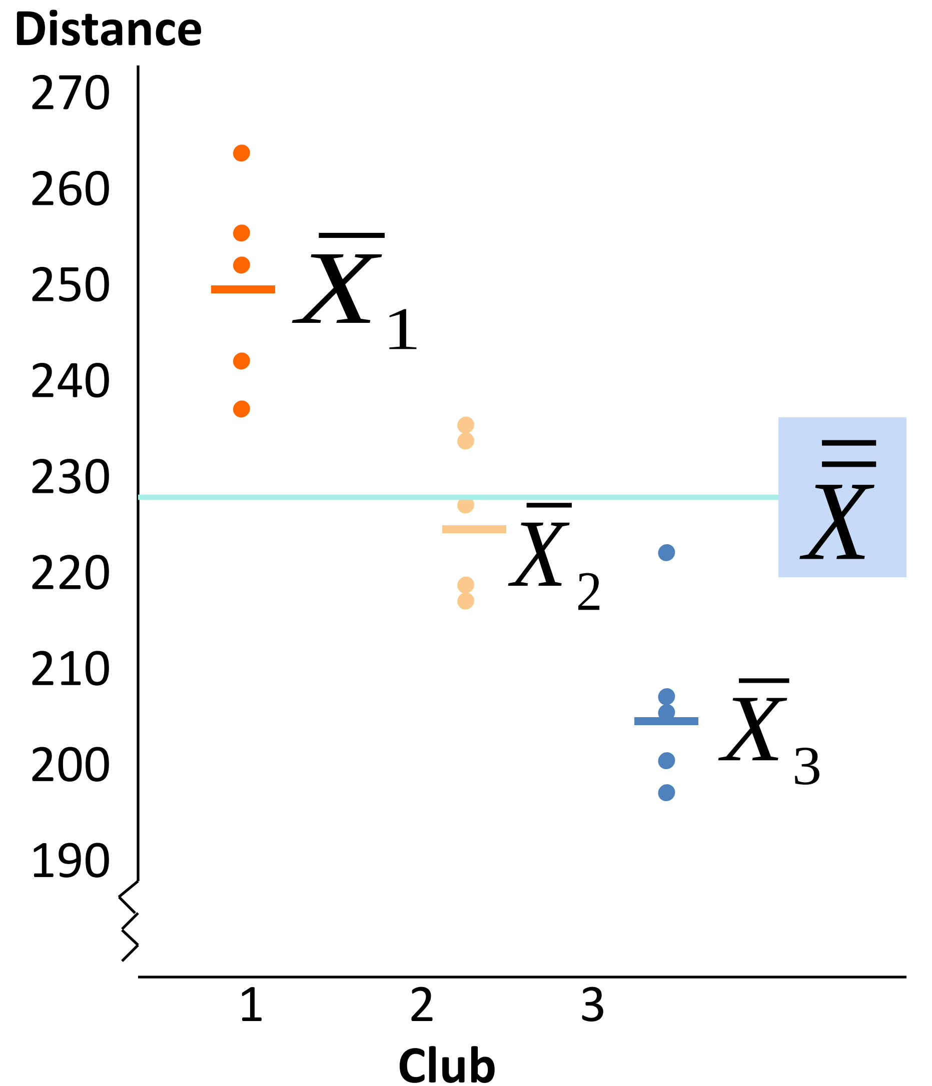
Club 1	Club 2	Club 3
254	234	200
263	218	222
241	235	197
237	227	206
251	216	204

One-Way ANOVA Example: Scatter Plot

Club 1	Club 2	Club 3
254	234	200
263	218	222
241	235	197
237	227	206
251	216	204

$\bar{X}_1 = 249.2$	$\bar{X}_2 = 226.0$	$\bar{X}_3 = 205.8$

$\bar{\bar{X}} = 227.0$



One-Way ANOVA Example Computations

Club 1	**Club 2**	**Club 3**
254	234	200
263	218	222
241	235	197
237	227	206
251	216	204

$\bar{X}_1 = 249.2$ $n_1 = 5$

$\bar{X}_2 = 226.0$ $n_2 = 5$

$\bar{X}_3 = 205.8$ $n_3 = 5$

$\bar{\bar{X}} = 227.0$ $n = 15$

$c = 3$

$SSA = 5\,(249.2 - 227)^2 + 5\,(226 - 227)^2 + 5\,(205.8 - 227)^2 = 4{,}716.4$

$SSW = (254 - 249.2)^2 + (263 - 249.2)^2 + \cdots + (204 - 205.8)^2 = 1{,}119.6$

$MSA = 4{,}716.4 \,/\, (3\text{-}1) = 2{,}358.2$

$MSW = 1{,}119.6 \,/\, (15\text{-}3) = 93.3$

$$F_{STAT} = \frac{2{,}358.2}{93.3} = 25.275$$

One-Way ANOVA Example Solution

$H_0: \mu_1 = \mu_2 = \mu_3$

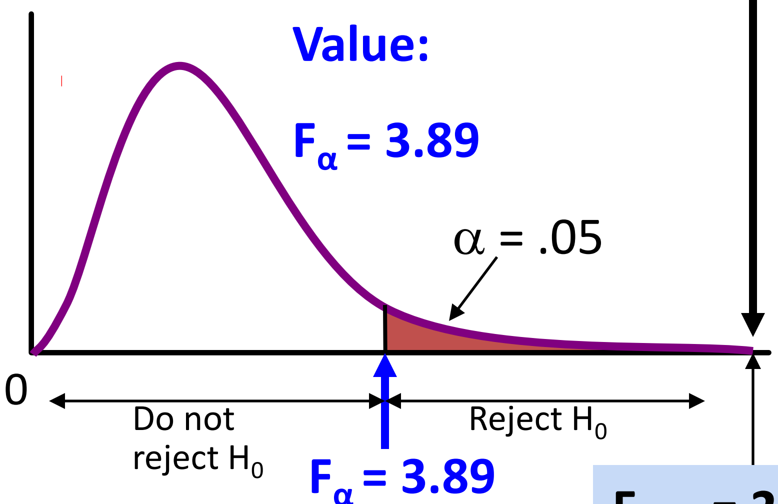
$H_1: \mu_j$ not all equal

$\alpha = 0.05$

$df_1 = 2 \quad df_2 = 12$

Critical Value:

$F_\alpha = 3.89$



$F_{STAT} = 25.275$

Test Statistic:

$$F_{STAT} = \frac{MSA}{MSW} = \frac{2358.2}{93.3} = 25.275$$

Decision:

Reject H_0 at $\alpha = 0.05$

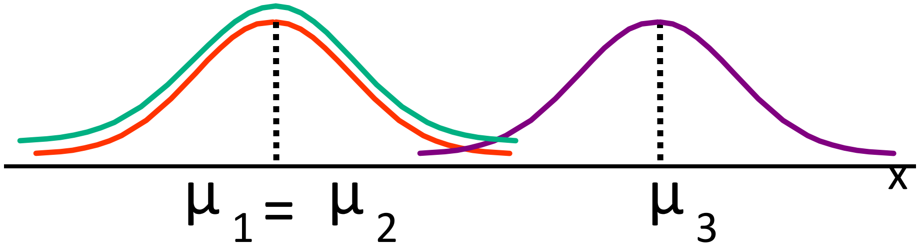
Conclusion:

There is evidence that at least one μ_j differs from the rest

TUKEY-KRAMER OF ANOVA

The Tukey-Kramer Procedure

- Tells which population means are significantly different
  - e.g.: $\mu_1 = \mu_2 \neq \mu_3$
  - Done after rejection of equal means in ANOVA
- Allows paired comparisons
  - Compare absolute mean differences with critical range



Tukey-Kramer Critical Range

$$\textit{Critical Range} = Q_{\alpha}\sqrt{\frac{MSW}{2}\left(\frac{1}{n_j}+\frac{1}{n_{j'}}\right)}$$

where:

Q_{α} = Upper Tail Critical Value from Studentized Range Distribution with c and n - c degrees of freedom (see appendix E.7 table)

MSW = Mean Square Within

n_j and $n_{j'}$ = Sample sizes from groups j and j’

The Tukey-Kramer Procedure: Example

Club 1	Club 2	Club 3
254	234	200
263	218	222
241	235	197
237	227	206
251	216	204

1. Compute absolute mean differences:

$$|\bar{x}_1 - \bar{x}_2| = |249.2 - 226.0| = 23.2$$

$$|\bar{x}_1 - \bar{x}_3| = |249.2 - 205.8| = 43.4$$

$$|\bar{x}_2 - \bar{x}_3| = |226.0 - 205.8| = 20.2$$

2. Find the Q_α value from the table in appendix E.7 with $c = 3$ and $(n - c) = (15 - 3) = 12$ degrees of freedom:

$$Q_\alpha = 3.77$$

The Tukey-Kramer Procedure: Example

3. Compute Critical Range:

$$\textit{Critical Range} = Q_\alpha \sqrt{\frac{MSW}{2}\left(\frac{1}{n_j}+\frac{1}{n_{j'}}\right)} = 3.77\sqrt{\frac{93.3}{2}\left(\frac{1}{5}+\frac{1}{5}\right)} = 16.285$$

4. Compare:

$$|\bar{x}_1 - \bar{x}_2| = 23.2$$

$$|\bar{x}_1 - \bar{x}_3| = 43.4$$

$$|\bar{x}_2 - \bar{x}_3| = 20.2$$

5. All of the absolute mean differences are greater than critical range. Therefore there is a significant difference between each pair of means at 5% level of significance. Thus, with 95% confidence we can conclude that the mean distance for club 1 is greater than club 2 and 3, and club 2 is greater than club 3.

ANOVA Assumptions

- Randomness and Independence
  - Select random samples from the c groups (or randomly assign the levels)
- Normality
  - The sample values for each group are from a normal population
- Homogeneity of Variance
  - All populations sampled from have the same variance
  - Can be tested with Levene’s Test

EXERCISE

11.7 (cont'd)

The Computer Anxiety Rating Scale (CARS) measures an individual's level of computer anxiety, on a scale from 20 (no anxiety) to 100 (highest level of anxiety). Researchers at Miami University administered CARS to 172 business students. One of the objectives of the study was to determine whether there are differences in the amount of computer anxiety experienced by students with different majors. They found the following:

11.7 (cont'd)

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F
Among majors	5	3,172		
Within majors	166	21,246		
Total	171	24,418		

Major	n	Mean
Marketing	19	44.37
Management	11	43.18
Other	14	42.21
Finance	45	41.8
Accountancy	36	37.56
MIS	47	32.21

11.7

a. Complete the ANOVA summary table.
b. At the 0.05 level of significance, is there evidence of a difference in the mean computer anxiety experienced by different majors?
c. If the results in (b) indicate that it is appropriate, use the Tukey-Kramer procedure to determine which majors differ in mean computer anxiety. Discuss your findings.

11.10 (cont'd)

A manufacturer of pens has hired an advertising agency to develop an advertising campaign for the upcoming holiday season. To prepare for this project, the research director decides to initiate a study of the effect of advertising on product perception. An experiment is designed to compare five different advertisements. Advertisement *A* greatly undersells the pen's characteristics. Advertisement *B* slightly undersells the pen's characteristics. Advertisement *C* slightly oversells the pen's characteristics. Advertisement *D* greatly oversells the pen's characteristics. Advertisement *E* attempts to correctly state the pen's characteristics.

11.10 (cont'd)

A sample of 30 adult respondents, taken from a larger focus group, is randomly assigned to the five advertisements (so that there are 6 respondents to each). After reading the advertisement and developing a sense of "product expectation," all respondents unknowingly receive the same pen to evaluate. The respondents are permitted to test the pen and the plausibility of the advertising copy. The respondents are then asked to rate the pen from 1 to 7 (lowest to highest) on the product characteristic scales of appearance, durability, and writing performance. The *combined* scores of three ratings (appearance, durability, and writing performance) for the 30 respondents (stored in Pen) are as follows

11.10 (cont'd)

A	B	C	D	E
15	16	8	5	12
18	17	7	6	19
17	21	10	13	18
19	16	15	11	12
19	19	14	9	17
20	17	14	10	14

11.10

a. At the 0.05 level of significance, is there evidence of a difference in the mean rating of the pens following exposure to five advertisements?
b. If appropriate, determine which advertisements differ in mean ratings.
c. Which advertisement(s) should you use, and which advertisement(s) should you avoid? Explain.

11.12 (cont'd)

Integrated circuits are manufactured on silicon wafers through a process that involves a series of steps. An experiment was carried out to study the effect on the yield of using three methods in the cleansing step (coded to maintain confidentiality). The results are as follows:

11.12 (cont'd)

New method 1	New Method 2	Standard
38	29	31
34	35	23
38	34	38
34	20	29
19	35	32
28	37	30

11.12

a. At the 0.05 level of significance, is there evidence of a difference in the mean yield among the methods used in the cleansing steps?
b. If appropriate, determine which methods differ in mean yields.

THANK YOU