Business Statistic

Week 13

Chi-Square Test

Learning Objectives

In this chapter, you learn:

- How and when to use the chi-square test for contingency tables

χ^2 TEST FOR THE DIFFERENCE BETWEEN TWO PROPORTIONS

Contingency Tables

Contingency Tables

- Useful in situations comparing multiple population proportions
- Used to classify sample observations according to two or more characteristics
- Also called a cross-classification table.

Contingency Table Example

Left-Handed vs. Gender

Dominant Hand: Left vs. Right

Gender: Male vs. Female

- 2 categories for each variable, so this is called a 2 x 2 table
- Suppose we examine a sample of 300 children

Contingency Table Example

Sample results organized in a contingency table:

sample size = n = 300:

120 Females, 12 were left handed

180 Males, 24 were left handed

Gender	Hand Preference		
	Left	Right	
Female	12	108	120
Male	24	156	180
	36	264	300

χ^2 Test for the Difference Between Two Proportions

$H_0: \pi_1 = \pi_2$ (Proportion of females who are left handed is equal to the proportion of males who are left handed)

$H_1: \pi_1 \neq \pi_2$ (The two proportions are not the same – hand preference is not independent of gender)

- If H_0 is true, then the proportion of left-handed females should be the same as the proportion of left-handed males
- The two proportions above should be the same as the proportion of left-handed people overall

The Chi-Square Test Statistic

The Chi-square test statistic is:

$$\chi^2_{STAT} = \sum_{\textit{all}\ \text{cells}} \frac{(f_o - f_e)^2}{f_e}$$

- where:
  f_o = observed frequency in a particular cell
  f_e = expected frequency in a particular cell if H_0 is true

χ^2_{STAT} **for the 2 x 2 case has 1 degree of freedom**

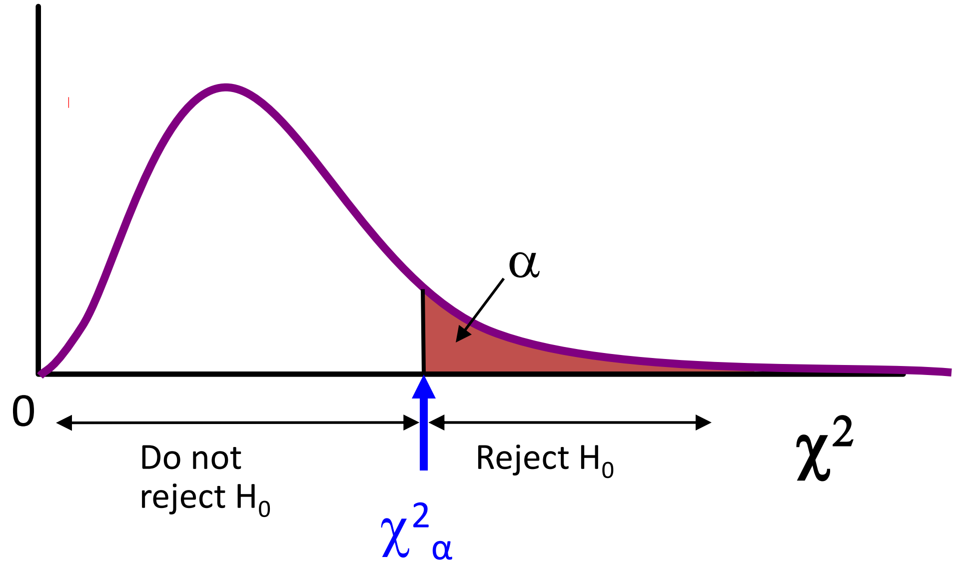
(Assumed: each cell in the contingency table has expected frequency of at least 5)

Decision Rule

The χ^2_{STAT} test statistic approximately follows a chi-squared distribution with one degree of freedom

Decision Rule:

If $\chi^2_{STAT} > \chi^2_{\alpha}$, reject H_0, otherwise, do not reject H_0



Computing the Average Proportion

The average proportion is:

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{X}{n}$$

120 Females, 12 were left handed

180 Males, 24 were left handed

Here:

$$\bar{p} = \frac{12 + 24}{120 + 180} = \frac{36}{300} = 0.12$$

i.e., based on all 300 children the proportion of left handers is 0.12, that is, 12%

Finding Expected Frequencies

- To obtain the expected frequency for left handed females, multiply the average proportion left handed (p) by the total_ number of females
- To obtain the expected frequency for left handed males, multiply the average proportion left handed (p) by the total number of males

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If the two proportions are equal, then

P(Left Handed | Female) = P(Left Handed | Male) = .12

i.e., we would expect **(.12)(120) = 14.4 females to be left handed**
(.12)(180) = 21.6 males to be left handed

Observed vs. Expected Frequencies

Gender	Hand Preference		
	Left	Right	
Female	Observed = 12 Expected = 14.4	Observed = 108 Expected = 105.6	120
Male	Observed = 24 Expected = 21.6	Observed = 156 Expected = 158.4	180
	36	264	300

The Chi-Square Test Statistic

Gender	Hand Preference		
	Left	Right	
Female	Observed = 12 Expected = 14.4	Observed = 108 Expected = 105.6	120
Male	Observed = 24 Expected = 21.6	Observed = 156 Expected = 158.4	180
	36	264	300

The test statistic is:

$$\chi^2_{STAT} = \sum_{\text{all cells}} \frac{(f_o - f_e)^2}{f_e}$$

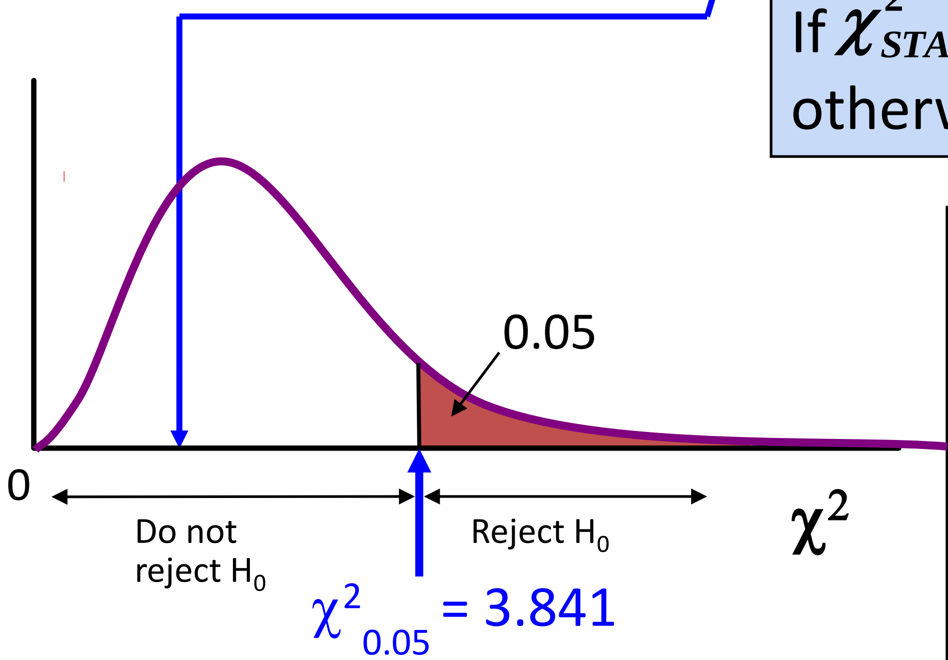
$$= \frac{(12-14.4)^2}{14.4} + \frac{(108-105.6)^2}{105.6} + \frac{(24-21.6)^2}{21.6} + \frac{(156-158.4)^2}{158.4} = 0.7576$$

Decision Rule

The test statistic is $\chi^2_{STAT} = 0.7576$; $\chi^2_{0.05}$ with 1 d.f. = 3.841

Decision Rule:

If $\chi^2_{STAT} > 3.841$, reject H_0, otherwise, do not reject H_0



Here,

$\chi^2_{STAT} = 0.7576 < \chi^2_{0.05} = 3.841$,

so we do not reject H_0 and conclude that there is not sufficient evidence that the two proportions are different at $\alpha = 0.05$

χ^2 TEST FOR DIFFERENCES AMONG MORE THAN TWO PROPORTIONS

χ^2 Test for Differences Among More Than Two Proportions

Extend the χ^2 test to the case with more than two independent populations:

H_0: $\pi_1 = \pi_2 = \cdots = \pi_c$

H_1: Not all of the π_j are equal ($j = 1, 2, \cdots, c$)

The Chi-Square Test Statistic

The Chi-square test statistic is:

$$\chi^2_{STAT} = \sum_{all\ \text{cells}} \frac{(f_o - f_e)^2}{f_e}$$

- Where:
  f_o = observed frequency in a particular cell of the 2 x c table
  f_e = expected frequency in a particular cell if H_0 is true

$$\chi^2_{STAT} \text{ for the 2 x c case has (2 - 1)(c - 1) = c - 1 degrees of freedom}$$

(Assumed: each cell in the contingency table has expected frequency of at least 1)

Computing the Overall Proportion

The overall proportion is:

$$\bar{p} = \frac{X_1 + X_2 + \Lambda + X_c}{n_1 + n_2 + \Lambda + n_c} = \frac{X}{n}$$

- Expected cell frequencies for the c categories are calculated as in the 2 x 2 case, and the decision rule is the same:

Decision Rule:
If $\chi^2_{STAT} > \chi^2_{\alpha}$ reject H_0, otherwise, do not reject H_0

Where χ^2_{α} is from the chi-squared distribution with c – 1 degrees of freedom

Example of χ^2 Test for Differences Among More Than Two Proportions

A University is thinking of switching to a trimester academic calendar. A random sample of 100 administrators, 50 students, and 50 faculty members were surveyed

Opinion	Administrators	Students	Faculty
Favor	63	20	37
Oppose	37	30	13
Totals	100	50	50

Using a 1% level of significance, which groups have a different attitude?

Chi-Square Test Results

H_0: $\pi_1 = \pi_2 = \pi_3$

H_1: Not all of the π_j are equal (j = 1, 2, 3)

Chi-Square Test: Administrators, Students, Faculty

	Admin	**Students**	**Faculty**	**Total**
Favor	63	20	37	120
	60	30	30	
Oppose	37	30	13	80
	40	20	20	
Total	100	50	50	200

Expected

Observed

$$\chi^2_{STAT} = 12.792 > \chi^2_{0.01} = 9.2103 \text{ so reject } H_0$$

The Marascuilo Procedure

- Used when the null hypothesis of equal proportions is rejected
- Enables you to make comparisons between all pairs
- Start with the observed differences, $p_j - p_{j'}$, for all pairs (for $j \neq j'$) then compare the absolute difference to a calculated critical range

The Marascuilo Procedure

- Critical Range for the Marascuilo Procedure:

$$\text{Critical range} = \sqrt{\chi^2_\alpha}\sqrt{\frac{p_j(1-p_j)}{n_j} + \frac{p_{j'}(1-p_{j'})}{n_{j'}}}$$

  - (Note: the critical range is different for each pairwise comparison)

- A particular pair of proportions is significantly different if

$$|p_j - p_{j'}| > \text{critical range for } j \text{ and } j'$$

Marascuilo Procedure Example

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Chi-Square Test Results

H_0: $\pi_1 = \pi_2 = \pi_3$

H_1: Not all of the π_j are equal (j = 1, 2, 3)

Chi-Square Test: Administrators, Students, Faculty

	Admin	Students	Faculty	Total
Favor	63	20	37	120
	60	30	30	
Oppose	37	30	13	80
	40	20	20	
Total	100	50	50	200

Expected

Observed

$$\chi^2_{STAT} = 12.792 > \chi^2_{0.01} = 9.2103 \text{ so reject } H_0$$

Marascuilo Procedure: Solution

Calculations In Excel:

compare

Marascuilo Procedure							
	Sample	Sample		Absolute	Std. Error	Critical	
Group	Proportion	Size	Comparison	Difference	of Difference	Range	Results
1	0.63	100	1 to 2	0.23	0.084445249	0.2563	Means are not different
2	0.4	50	1 to 3	0.11	0.078606615	0.2386	Means are not different
3	0.74	50	2 to 3	0.34	0.092994624	0.2822	Means are different

At 1% level of significance, there is evidence of a difference in attitude between students and faculty

Minitab does not do the Marascuilo procedure

χ^2 TEST OF INDEPENDENCE

χ^2 Test of Independence

Similar to the χ^2 test for equality of more than two proportions, but extends the concept to contingency tables with r rows and c columns

H_0: The two categorical variables are independent

(i.e., there is no relationship between them)

H_1: The two categorical variables are dependent

(i.e., there is a relationship between them)

χ^2 Test of Independence

The Chi-square test statistic is:

$$\chi^2_{STAT} = \sum_{all\ cells} \frac{(f_o - f_e)^2}{f_e}$$

- where:

  f_o = observed frequency in a particular cell of the r x c table

  f_e = expected frequency in a particular cell if H_0 is true

χ^2_{STAT} for the r x c case has $(r-1)(c-1)$ degrees of freedom

(Assumed: each cell in the contingency table has expected frequency of at least 1)

Expected Cell Frequencies

- Expected cell frequencies:

$$f_e = \frac{\text{row total} \times \text{column total}}{n}$$

Where:

row total = sum of all frequencies in the row

column total = sum of all frequencies in the column

n = overall sample size

Decision Rule

- The decision rule is

If $\chi^2_{STAT} > \chi^2_{\alpha}$, reject H_0,

otherwise, do not reject H_0

Where χ^2_{α} is from the chi-squared distribution with $(r - 1)(c - 1)$ degrees of freedom

Example

- The meal plan selected by 200 students is shown below:

Class Standing	Number of meals per week			Total
	20/week	10/week	none	
Fresh.	24	32	14	70
Soph.	22	26	12	60
Junior	10	14	6	30
Senior	14	16	10	40
Total	70	88	42	200

Example

- The hypothesis to be tested is:

H_0: Meal plan and class standing are independent
(i.e., there is no relationship between them)
H_1: Meal plan and class standing are dependent
(i.e., there is a relationship between them)

Example:
Expected Cell Frequencies

Observed:

Class Standing	Number of meals per week			Total
	20/wk	10/wk	none	
Fresh.	24	32	14	70
Soph.	22	26	12	60
Junior	10	14	6	30
Senior	14	16	10	40
Total	70	88	42	200

Expected cell frequencies if H_0 is true:

Class Standing	Number of meals per week			Total
	20/wk	10/wk	none	
Fresh.	24.5	30.8	14.7	70
Soph.	21.0	26.4	12.6	60
Junior	10.5	13.2	6.3	30
Senior	14.0	17.6	8.4	40
Total	70	88	42	200

Example for one cell:

$$f_e = \frac{\text{row total} \times \text{column total}}{n}$$

$$= \frac{30 \times 70}{200} = 10.5$$

Example: The Test Statistic

- The test statistic value is:

$$\chi^2_{STAT} = \sum_{all\ \text{cells}} \frac{(f_o - f_e)^2}{f_e}$$

$$= \frac{(24 - 24.5)^2}{24.5} + \frac{(32 - 30.8)^2}{30.8} + \Lambda + \frac{(10 - 8.4)^2}{8.4} = 0.709$$

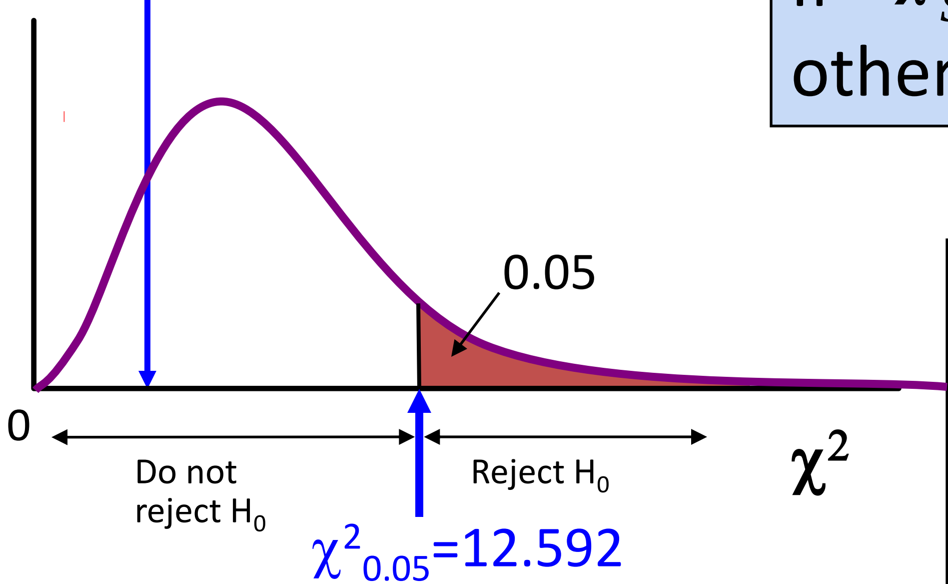
$\chi^2_{0.05}$ = 12.592 from the chi-squared distribution with (4 – 1)(3 – 1) = 6 degrees of freedom

Example: Decision and Interpretation

The test statistic is $\chi^2_{STAT} = 0.709$; $\chi^2_{0.05}$ with 6 d.f. $= 12.592$

Decision Rule:

If $\chi^2_{STAT} > 12.592$, reject H_0, otherwise, do not reject H_0



Here,
$\chi^2_{STAT} = 0.709 < \chi^2_{0.05} = 12.592$,
so do not reject H_0

Conclusion: there is not sufficient evidence that meal plan and class standing are related at $\alpha = 0.05$

EXERCISE

12.14

How do Americans feel about online ads tailored to their individual interests? A survey of 1,000 adult Internet users found that 55% of the 18 to 24 year olds, 59% of 25 to 34 year olds, 66% of 35 to 49 year olds, 77% of 50 to 64 year olds, and 82% of 65 to 89 year olds opposed such ads. Suppose that the survey was based on 200 respondents in each of five age groups: 18 to 24, 25 to 34, 35 to 49, 50 to 64, and 65 to 89. At the 0.05 level of significance, is there evidence of a difference among the age groups in the opposition to ads on web pages tailored to their interests?

12.16 (Cont’d)

More shoppers do the majority of their grocery shopping on Saturday than any other day of the week. However, is there a difference in the various age groups in the proportion of people who do the majority of their grocery shopping on Saturday? A study showed the results for the different age groups. The data were reported as percentages, and no sample sizes were given:

12.16

	USIA		
MAJOR SHOPPING DAY	Under 35	35–54	Over 54
Saturday	24%	28%	12%
A day other than Saturday	76%	72%	88%

Assume that 200 shoppers for each age group were surveyed. Is there evidence of a significant difference among the age groups with respect to major grocery shopping day? (Use $\alpha = 0.05$)

12.18

Is there a generation gap in music? A study reported that 45% of 16 to 29 year olds, 42% of 30 to 49 year olds, and 33% of 50 to 64 year olds often listened to rock music. Suppose that the study was based on a sample of 200 respondents in each group. Is there evidence of a significant difference among the age groups with respect to the proportion who often listened to rock music? (Use $\alpha = 0.05$)

12.24 (cont’d)

A large corporation is interested in determining whether a relationship exists between the commuting time of its employees and the level of stress-related problems observed on the job. A study of 116 workers reveals the following:

12.24 (cont'd)

Commuting Time	Stress Level			
	High	Moderate	Low	Total
Under 15 Min.	9	5	18	32
14-45 Min.	17	8	28	53
Over 45 Min.	18	6	7	31
Total	44	19	53	116

12.24

At the 0.01 level of significance, is there evidence of a significant relationship between commuting time and stress level?

THANK YOU