Statistic for Business

Week 2

Numerical Descriptive Measures

Agenda

Time	Activity
90 minutes	Central Tendency
60 minutes	Variation and Shape
30 minutes	Exploring Numerical Data

Objectives

By the end of this class, student should be able to understand:

- How to measures central tendency in statistics
- How to interpret those central tendency measurements

Numerical Descriptive Measures

Central Tendency

Variation and Shape

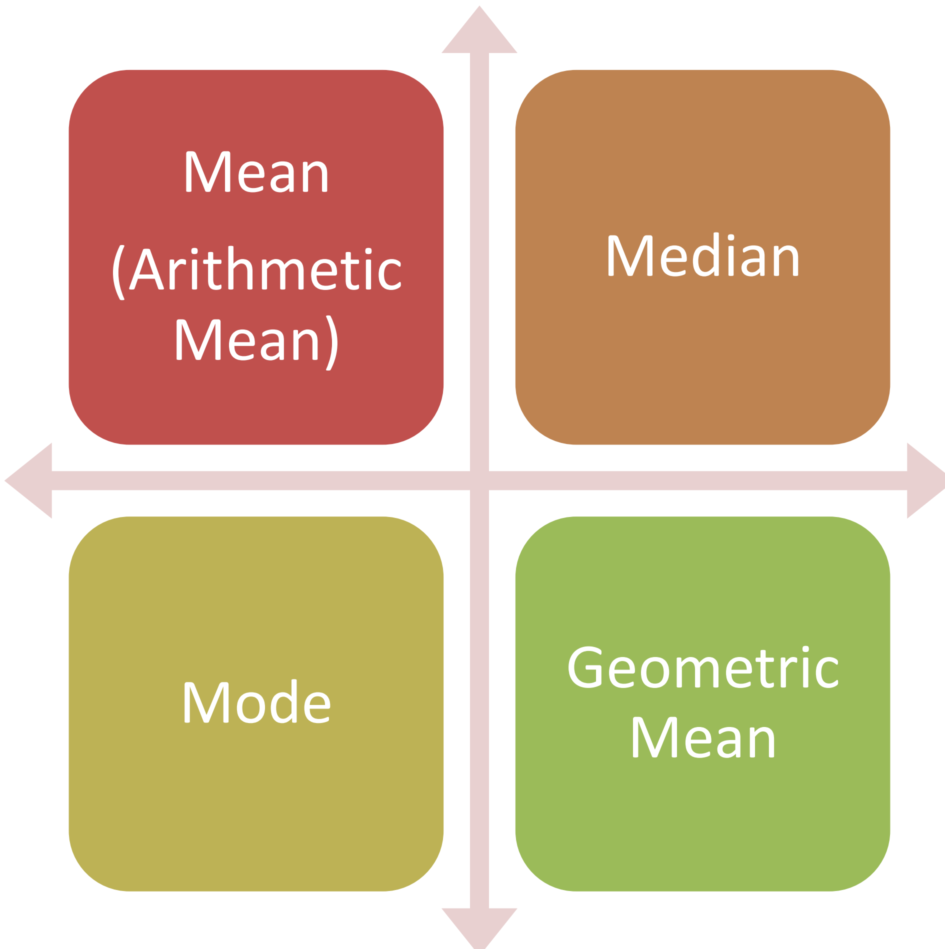
Exploring Numerical Data

Numerical Descriptive Measures for a Population

The Covariance and The Coefficient of Correlation

CENTRAL TENDENCY

Central Tendency



Mean

Consider this height data:

160 157 162 170 168 174 156 173 157

What is the mean height?

Mean

Pronounced x-bar

The i^th value

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{X_1 + X_2 + \Lambda + X_n}{n}$$

Sample size

Observed values

Mean

How about this data of business statistic's students monthly spending:

Monthly Spending	Frequency
less than Rp. 500.000	2
Rp. 500.000 but less than Rp. 1.000.000	7
Rp. 1.000.000 but less than Rp. 1.500.000	13
Rp. 1.500.000 but less than Rp. 2.000.000	5

What is the MEAN?

Mean

In this case we can only **ESTIMATE the MEAN**…

Spending	Frequency
less than Rp. 500.000	2
Rp. 500.000 but less than Rp. 1.000.000	7
Rp. 1.000.000 but less than Rp. 1.500.000	13
Rp. 1.500.000 but less than Rp. 2.000.000	5

Keyword: “MIDPOINTS”

Estimated Mean

Midpoint	Frequency	Mid * f
250000	2	500000
750000	7	5250000
1250000	13	16250000
1750000	5	8750000
Total	27	30750000

$$Estimated\ Mean = \frac{30750000}{27} = 1138888.89$$

Mean

The following is "Student A" Score:

Course	Credits	Score
Business Math	3	60
English	2	80
Organization Behavior	3	100
Statistics	4	90
Operation Management	3	70

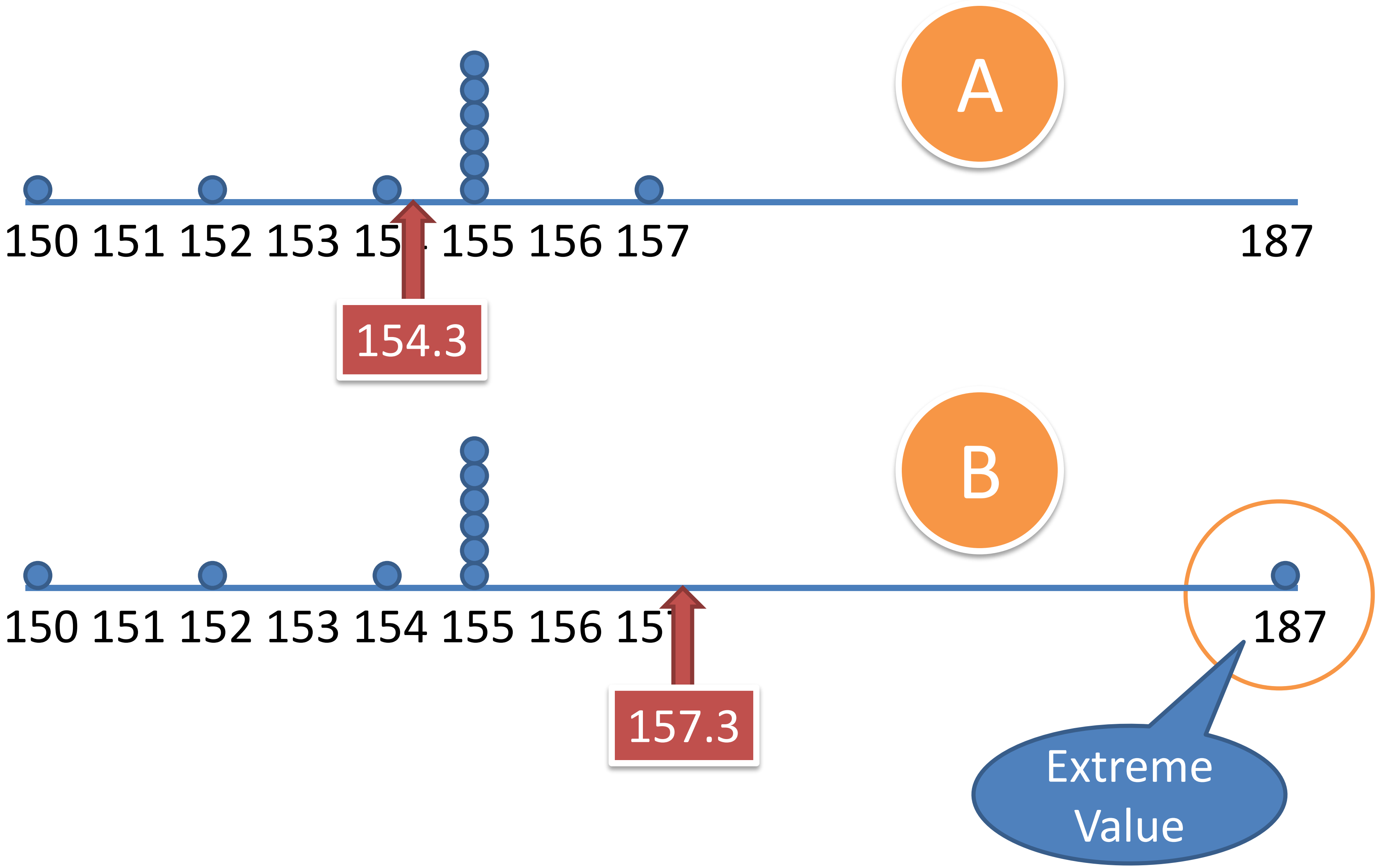
What is the average score of "Student A"?

Mean

Consider these two sets of data:

A	150 152 154 155 155 155 155 155 155 157	Mean?
B	150 152 154 155 155 155 155 155 155 187	Mean?

Mean


A
150 151 152 153 154 155 156 157
187
154.3
B
150 151 152 153 154 155 156 157
187
157.3
Extreme
Value

It is DANGEROUS
to ONLY use
MEAN in
describing a data

Median

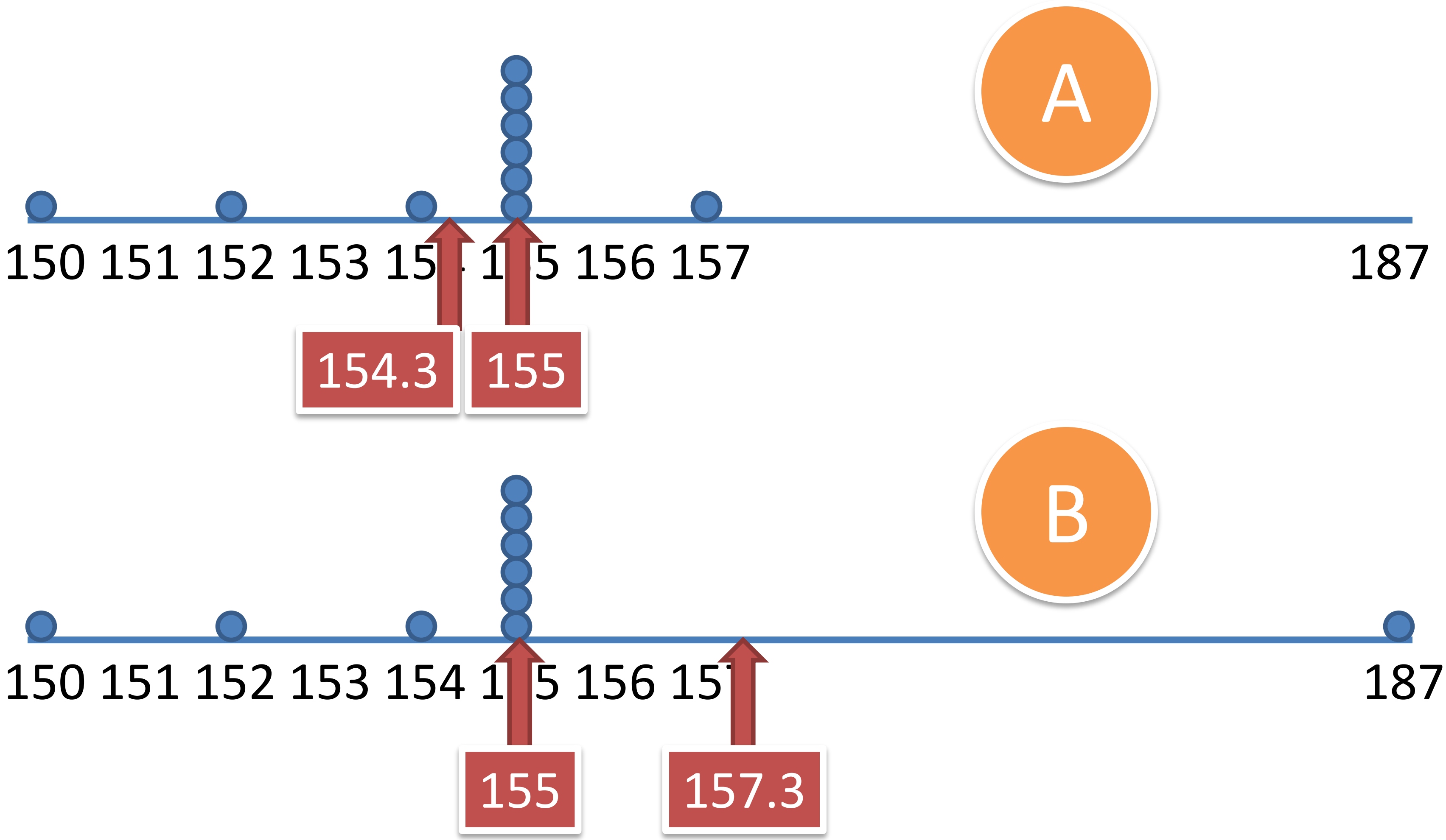
$$\text{Median position} = \frac{n+1}{2} \text{ position in the ordered data}$$

Median

Consider these two sets of data:

A	150 152 154 155 155 155 155 155 155 157	Median?
B	150 152 154 155 155 155 155 155 155 187	Median?

Median



A
150 151 152 153 156 157
187
154.3
155
B
150 151 152 153 154 156
187
155
157.3

Median

What is the median of this height data:

160 157 162 170 168 174 156 173 157

How about this data:

160 157 162 170 168 174 156 173 157 150

Median

How about this data of business statistic's students monthly spending:

Monthly Spending	Frequency
less than Rp. 500.000	2
Rp. 500.000 but less than Rp. 1.000.000	7
Rp. 1.000.000 but less than Rp. 1.500.000	13
Rp. 1.500.000 but less than Rp. 2.000.000	5

What is the MEDIAN?

Median

The MEDIAN group of monthly spending is Rp. 1.000.000 but less than Rp. 1.500.000

Or ESTIMATE the MEDIAN!!

Estimated Median

Monthly Spending	Frequency
less than Rp. 500.000	2
Rp. 500.000 but less than Rp. 1.000.000	7
Rp. 1.000.000 but less than Rp. 1.500.000	13
Rp. 1.500.000 but less than Rp. 2.000.000	5

Estimated Median = Rp. 1.173.076,92

Estimated Median

$$\textbf{Estimated Median} = L + \frac{(n/2) - cf_b}{f_m} \times w$$

where:

- **L** is the lower class boundary of the group containing the median
- **n** is the total number of data
- $\mathbf{cf_b}$ is the cumulative frequency of the groups before the median group
- $\mathbf{f_m}$ is the frequency of the median group
- **w** is the group width

Mode

What is the mode of this height data:

160 157 162 170 168 174 156 173 157

How about this data:

160 157 162 170 168 174 156 173 150

Mode

How about this data of business statistic's students monthly spending:

Spending	Frequency
less than Rp. 500.000	2
Rp. 500.000 but less than Rp. 1.000.000	7
Rp. 1.000.000 but less than Rp. 1.500.000	13
Rp. 1.500.000 but less than Rp. 2.000.000	5

What is the MODE?

Mode

The MODAL group of monthly spending is Rp. 1.000.000 but less than Rp. 1.500.000

But the actual **Mode** may not even be in that group!

Mode

Without the raw data we don't really know…

However, we can ESTIMATE the MODE

Estimated Mode

Spending	Frequency
less than Rp. 500.000	2
Rp. 500.000 but less than Rp. 1.000.000	7
Rp. 1.000.000 but less than Rp. 1.500.000	13
Rp. 1.500.000 but less than Rp. 2.000.000	5

Estimated Mode = Rp. 1.214.285,72

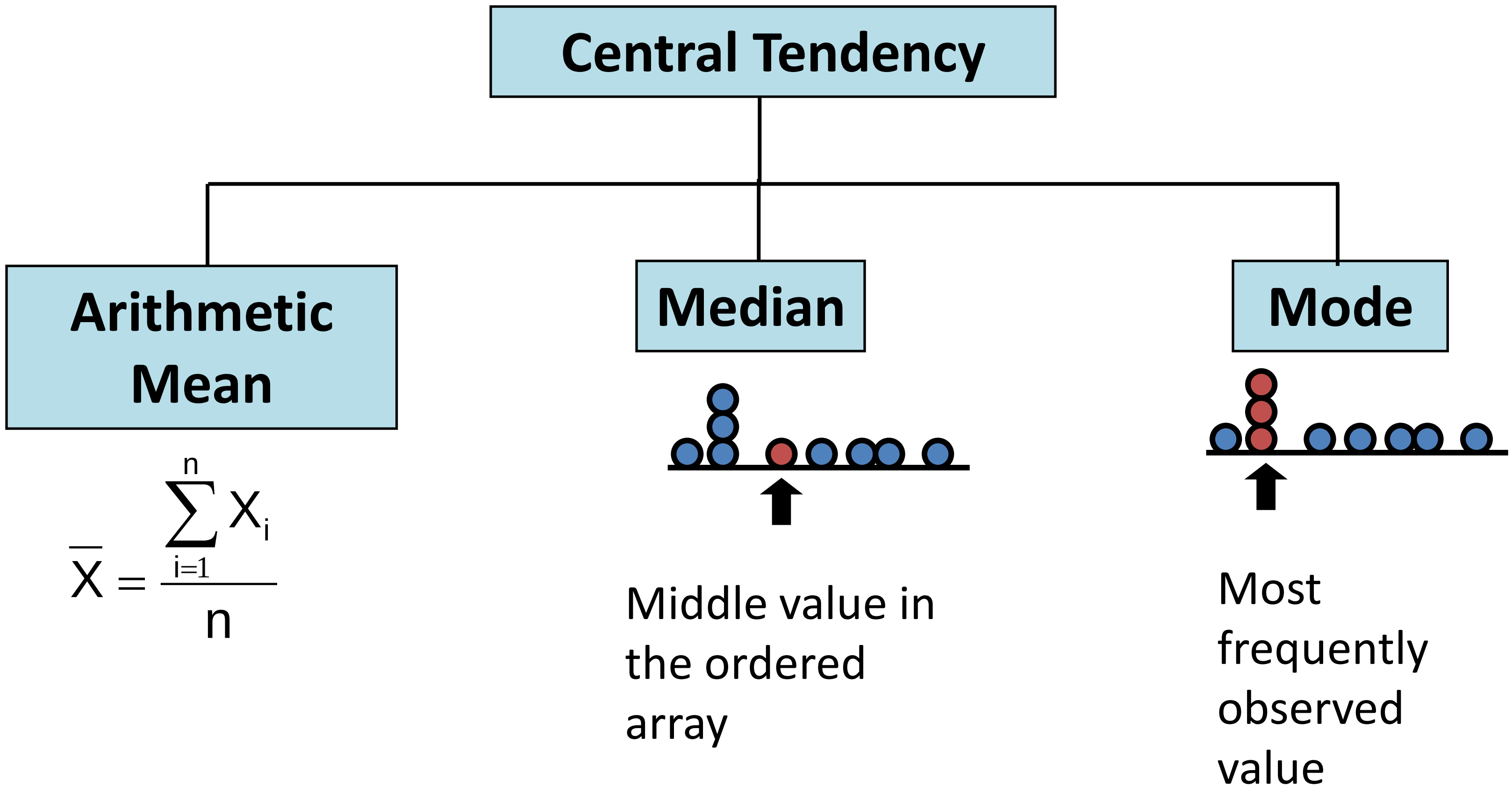
Estimated Mode

$$\textbf{Estimated Mode} = L + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \times w$$

where:

- L is the lower class boundary of the modal group
- f_{m-1} is the frequency of the group before the modal group
- f_m is the frequency of the modal group
- f_{m+1} is the frequency of the group after the modal group
- w is the group width

Central Tendency



EXERCISE

3.10

This is the data of the amount that sample of nine customers spent for lunch ($) at a fast-food restaurant:

4.20 5.03 5.86 6.45 7.38 7.54 8.46 8.47 9.87

Compute the mean and median.

3.12

The following data is the overall miles per gallon (MPG) of 2010 small SUVs:

24	23	22	21	22	22	18	18	26
26	26	19	19	19	21	21	21	21
21	18	29	21	22	22	16	16	

Compute the median and mode.

GEOMETRIC MEAN

Compounding Data

Interest Rate

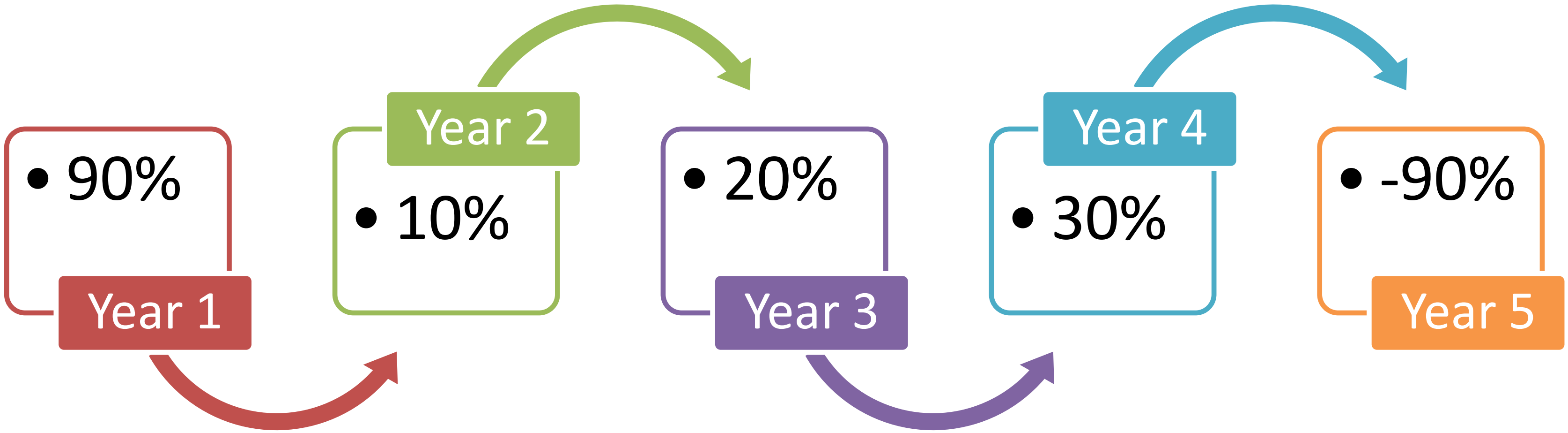
Growth Rate

Return Rate

Compounding Data

Suppose you have invested your savings in the stock market for five years. If your returns each year were 90%, 10%, 20%, 30% and -90%, what would your average return be during this period?

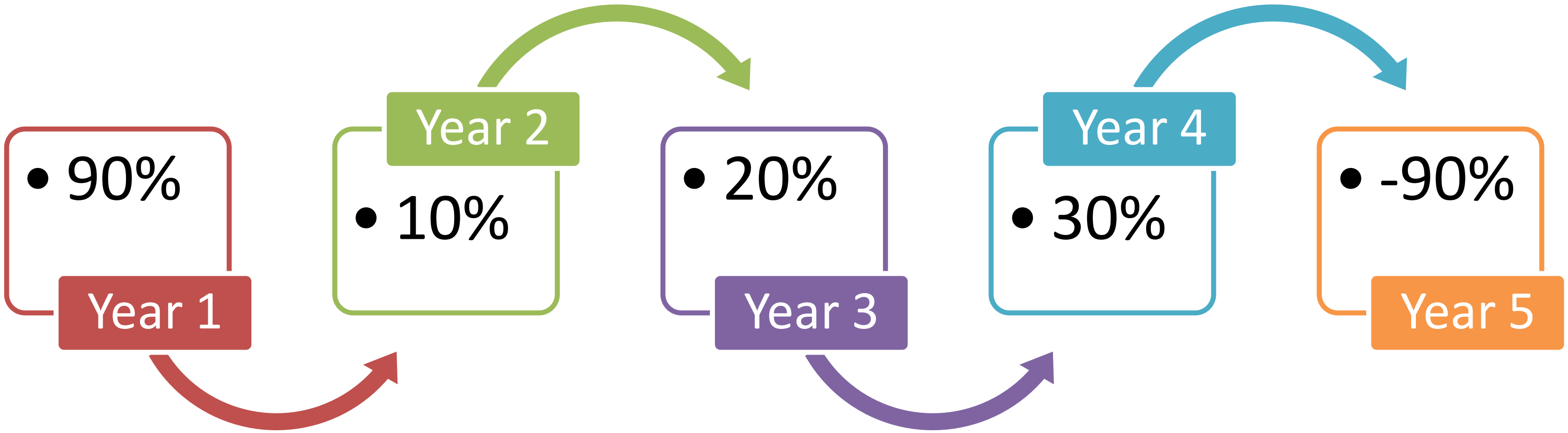
Compounding Data



If we use arithmetic mean in this case

The average return during this period = **12%**

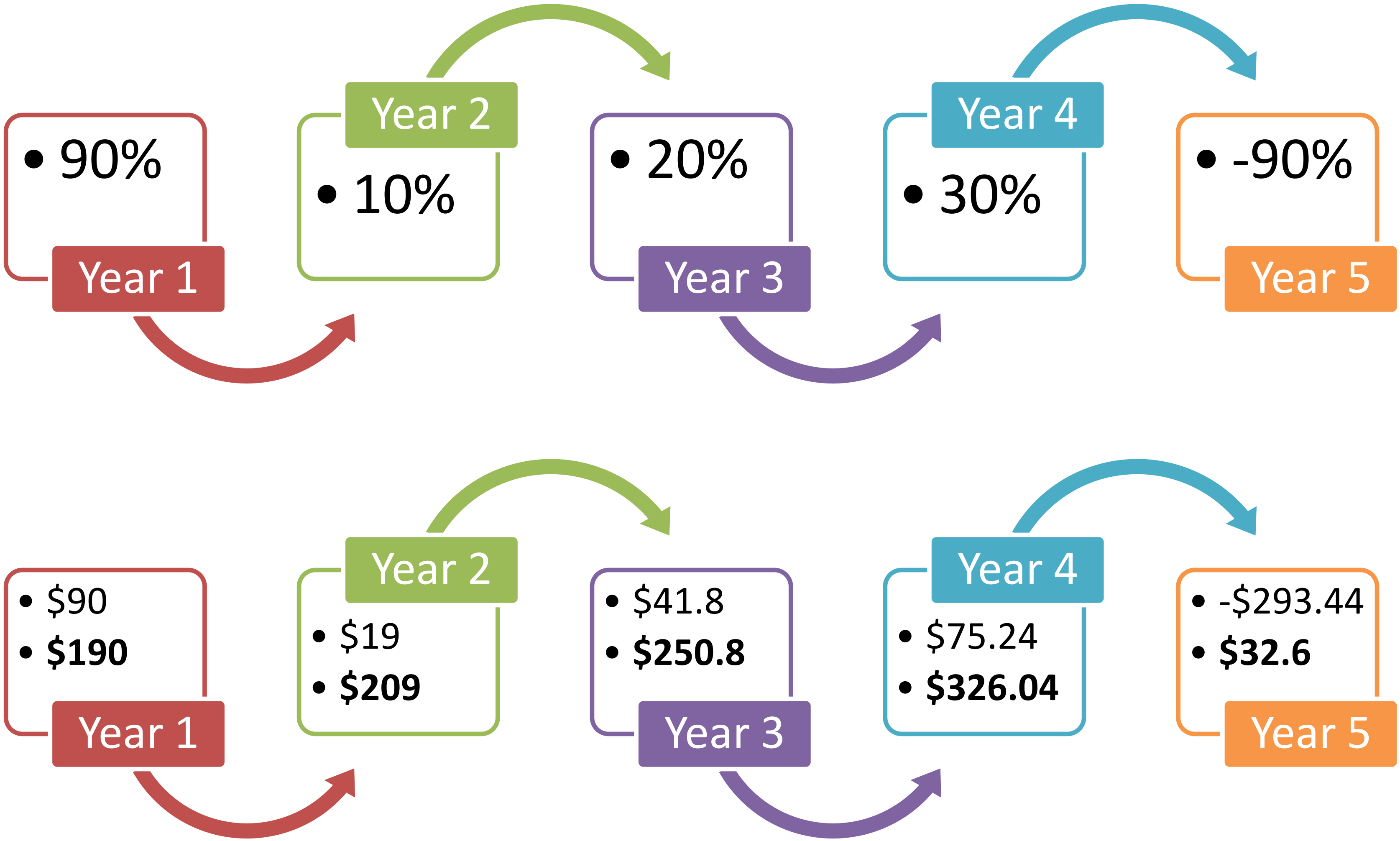
Compounding Data



Let say that you invest $100 in year 0

How much your stocks worth in year 5?

Compounding Data



Geometric Mean

$$GM = \sqrt[5]{1.9 \times 1.1 \times 1.2 \times 1.3 \times 0.1} - 1$$

$$GM = -20.08\%$$

This is called **geometric mean rate of return**

Measure of Central Tendency For The Rate Of Change Of A Variable Over Time: The Geometric Mean & The Geometric Rate of Return

- Geometric mean
  - Used to measure the rate of change of a variable over time

$$\overline{X}_G = (X_1 \times X_2 \times \Lambda \times X_n)^{1/n}$$

- Geometric mean rate of return
  - Measures the status of an investment over time

$$\overline{R}_G = [(1+R_1)\times(1+R_2)\times\Lambda \times(1+R_n)]^{1/n} - 1$$

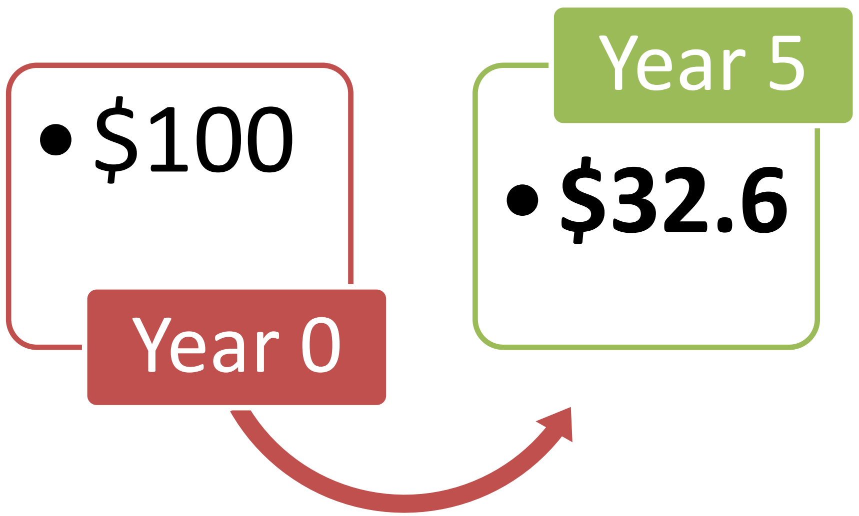
  - Where R_i is the rate of return in time period i

Geometric Mean

$$GM = \sqrt[n]{\frac{End\ of\ Period\ Value}{Beginning\ of\ period\ Value}} - 1$$

Geometric Mean

Lets reconsider the previous problem. We knew that we invest $100 in year 0 (zero). However, by the end of year 5 the value of the stock became $32.6. Calculate the annual average return!



Geometric Mean

$$GM = \sqrt[5]{\frac{32.6}{100}} - 1$$

$$GM = -20.08\%$$

- This value consistent with what we found earlier

Population of West Java

Population of West Java:

- Year 2000: 35.729.537
- Year 2010: 43.053.732

Population growth rate per year?

3.22

In 2006-2009, the value of precious metals changed rapidly. The data in the following table represent the total rate of return (in percentage) for platinum, gold, an silver from 2006 through 2009:

Year	Platinum	Gold	Silver
2009	62.7	25.0	56.8
2008	-41.3	4.3	-26.9
2007	36.9	31.9	14.4
2006	15.9	23.2	46.1

3.22

a. Compute the geometric mean rate of return per year for platinum, gold, and silver from 2006 through 2009.
b. What conclusions can you reach concerning the geometric mean rates of return of the three precious metals?

VARIATION AND SHAPE

Variation and Shape

- Range
- Variance and Standard Deviation
- Coefficient of Variation
- Z Scores
- Shape

Review on Central Tendency

Consider this data:

160 157 162 170 168 174 156 173 157 150

What is the mean, median, and mode?

Range

Consider this data:

160 157 162 170 168 174 156 173 157 150

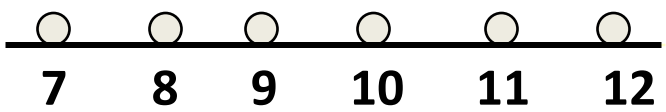
What is the Range?

Range

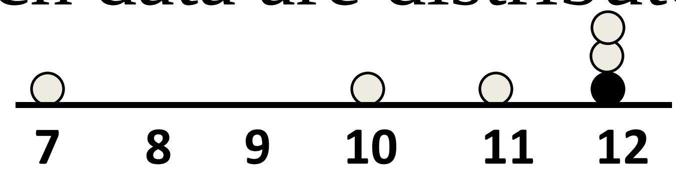
$$Range = X_{\max} - X_{\min}$$

Measures of Variation:
Why The Range Can Be Misleading

- Ignores the way in which data are distributed



Range = 12 - 7 = 5



Range = 12 - 7 = 5

- Sensitive to outliers

1,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,3,3,3,3,4,**5**

Range = 5 - 1 = 4

1,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,3,3,3,3,4,**120**

Range = 120 - 1 = 119

Variance and Standard Deviation

Deviation

Let's see this data again:

160 157 162 170 168 174 156 173 157 150

What is the mean?

Mean = 162.7

Deviation

Data	Deviation
160	-2.7
157	-5.7
162	-0.7
170	7.3
168	5.3
174	11.3
156	-6.7
173	10.3
157	-5.7
150	-12.7

$$Deviation = X_i - \overline{X}$$

=156-162.7

Variance and Standard Deviation

Data	Deviation	(Dev)^2
160	-2.7	7.29
157	-5.7	32.49
162	-0.7	0.49
170	7.3	53.29
168	5.3	28.09
174	11.3	127.69
156	-6.7	44.89
173	10.3	106.09
157	-5.7	32.49
150	-12.7	161.29

Sum of Squares = 594.1

Variance and Standard Deviation

Sample size (n) = 10

$$\text{Variance} = \frac{594.1}{10 - 1} = 66.01$$

$$\text{SD} = \sqrt{66.01} = 8.125$$

Variance and Standard Deviation

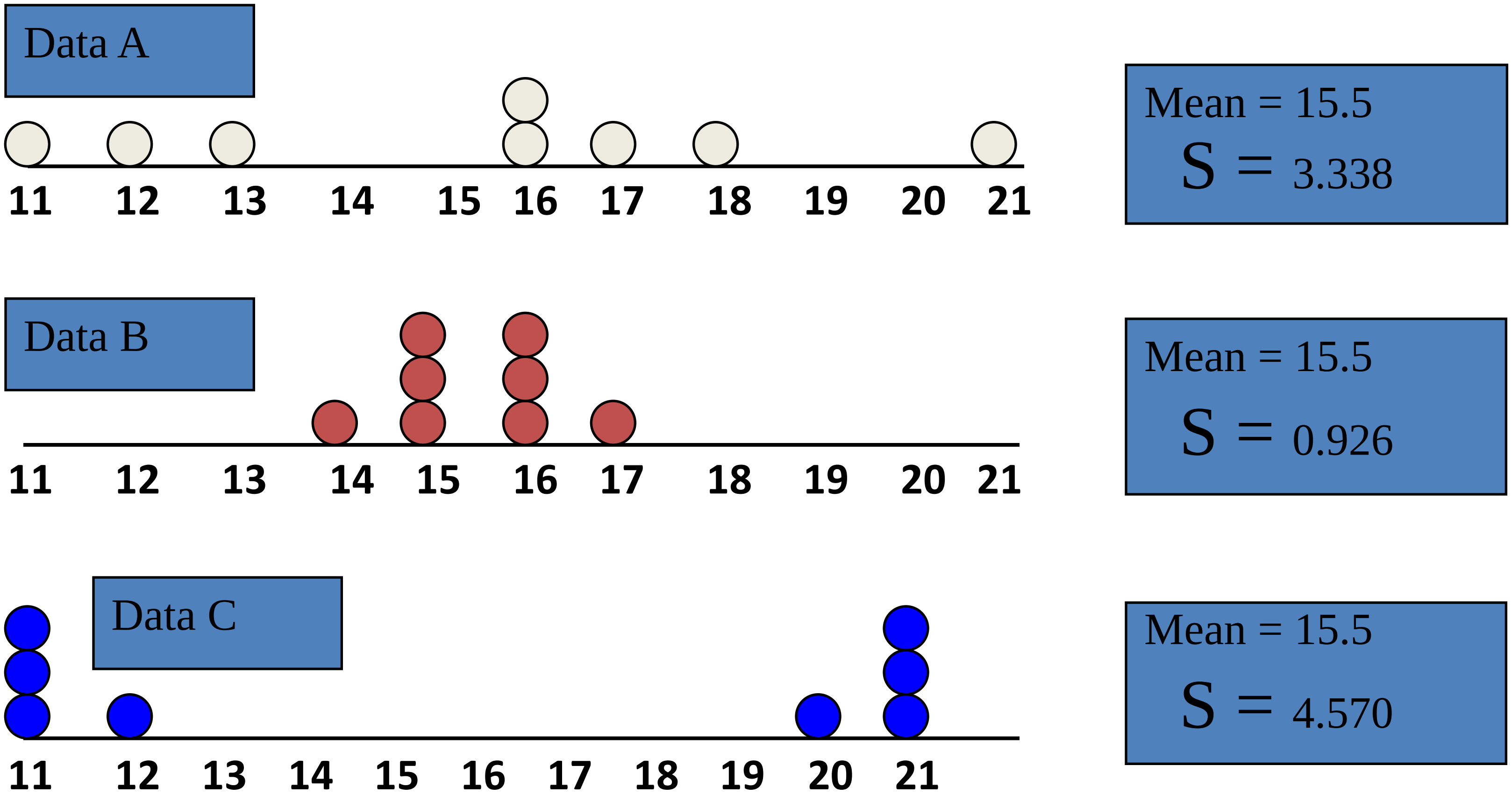
- Sample

$$S^2 = \frac{\sum_{i=1}^{n} \left(X_i - \overline{X} \right)^2}{n-1}$$

- Population

$$\sigma^2 = \frac{\sum_{i=1}^{n} \left(X_i - \mu \right)^2}{N}$$

Measures of Variation: Comparing Standard Deviations



Standard Deviation

How about this data of business statistic's students monthly spending:

Monthly Spending	Frequency
less than Rp. 500.000	2
Rp. 500.000 but less than Rp. 1.000.000	7
Rp. 1.000.000 but less than Rp. 1.500.000	13
Rp. 1.500.000 but less than Rp. 2.000.000	5

What is the STANDARD DEVIATION?

Standard Deviation

How about this data of business statistic's students month[illegible]

Mon[illegible]	[illegible]ency
less than Rp. 50[illegible]	[illegible]
Rp. 500.000 bu[illegible]	[illegible]
Rp. 1.000.000 but le[illegible] Rp. 1.500.000	13
Rp. 1.500.000 but l[illegible] than Rp. 2.000.000	5

E.S.T.I.M.A.T.I.O.N

What is the STANDARD DEVIATION?

Estimated Standard Deviation

Midpoint	Frequency	Dev^2	(Dev^2)*f
250000	2	790123456790.12	1580246913580.25
750000	7	151234567901.24	1058641975308.64
1250000	13	12345679012.35	160493827160.49
1750000	5	373456790123.46	1867283950617.28
Total	**27**		**4666666666666.67**

$$Variance = \frac{4666666666666.67}{27} = 172839506172.84$$

$$SD = \sqrt{172839506172.84} = 415739.71$$

THE COEFFICIENT OF VARIATION

The Coefficient of Variation

- Height
- Weight

The Coefficient of Variation

Let's see this height data again:

160 157 162 170 168 174 156 173 157 150

What is the mean and standard deviation

Mean = 162.7 and SD = 8.125

The Coefficient of Variation

Students with height before is weighted as follows:

50	55	57	52	55
69	60	65	71	70

What is mean and standard deviation?

Mean = 60.4 and SD = 7.8

The Coefficient of Variation

	Height	Weight
Mean	162.7	60.4
SD	8.125	7.8

Which one has more variability?

Coefficient of Variation:

CV_{Height} = 4.99%

CV_{Weight}= 12.92%

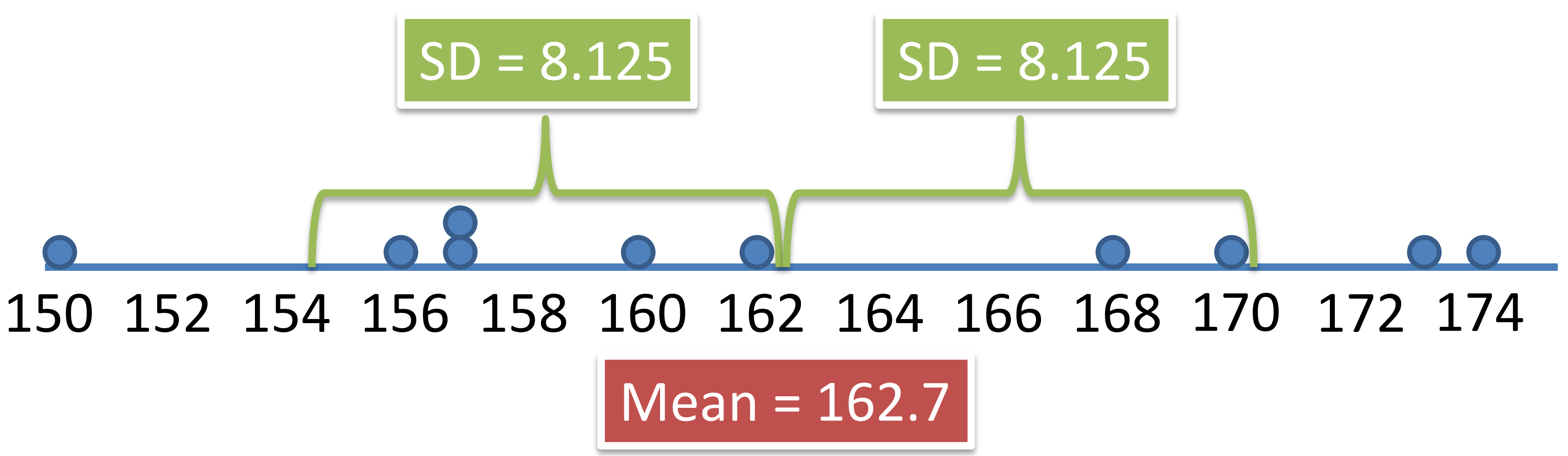
The Coefficient of Variation

$$CV = \left(\frac{SD}{\overline{X}} \right) .100\%$$

Locating Extreme Outliers: Z Score

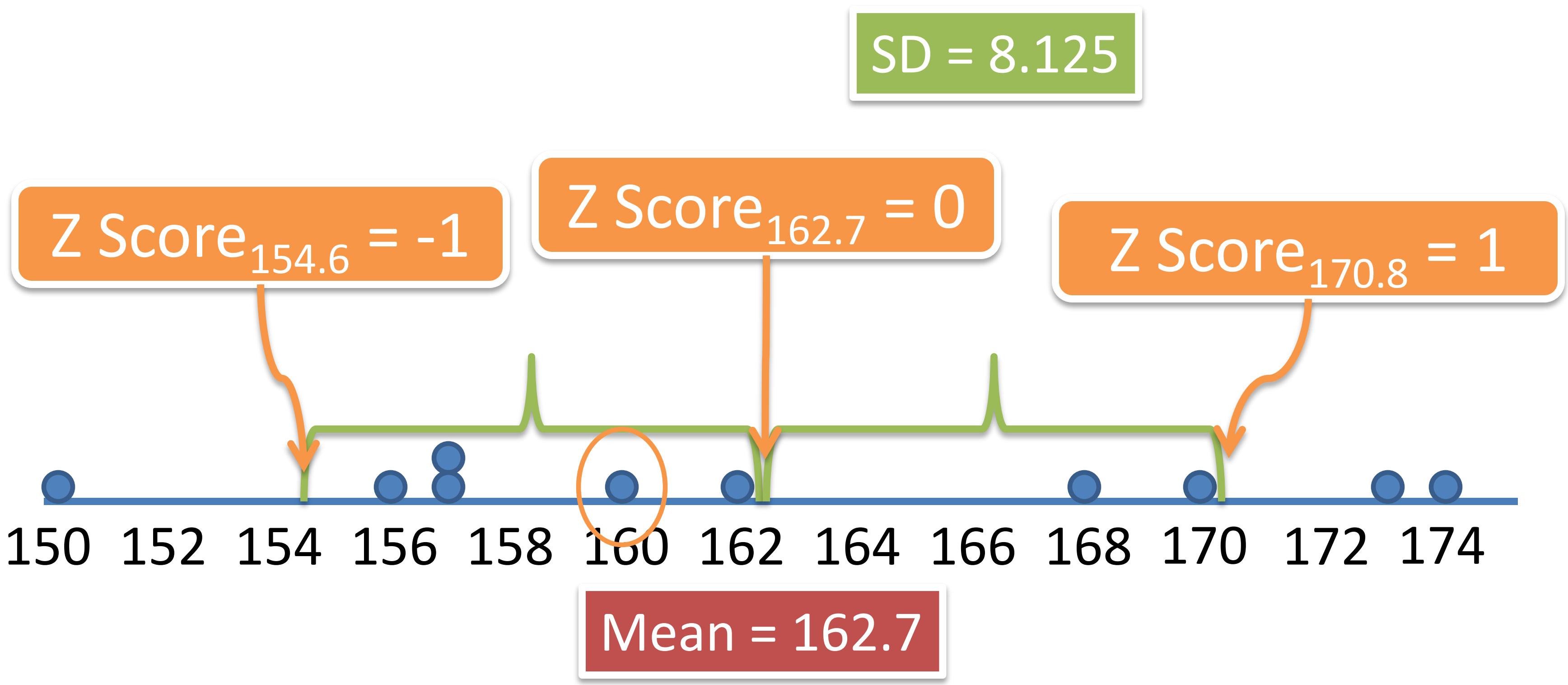
Let's see this height data again:

160 157 162 170 168 174 156 173 157 150



Locating Extreme Outliers: Z Score

Therefore, Z Score for 160 is?



Locating Extreme Outliers: Z Scores

Let's see this height data again:

160 157 162 170 168 174 156 173 157 150

What is the Z Scores of 160, 174, 168 and 150?

$Z_{160} = -0.33$, $Z_{174} = 1.39$, $Z_{168} = 0.65$, and
$Z_{150} = -0.56$

Locating Extreme Outliers: Z Score

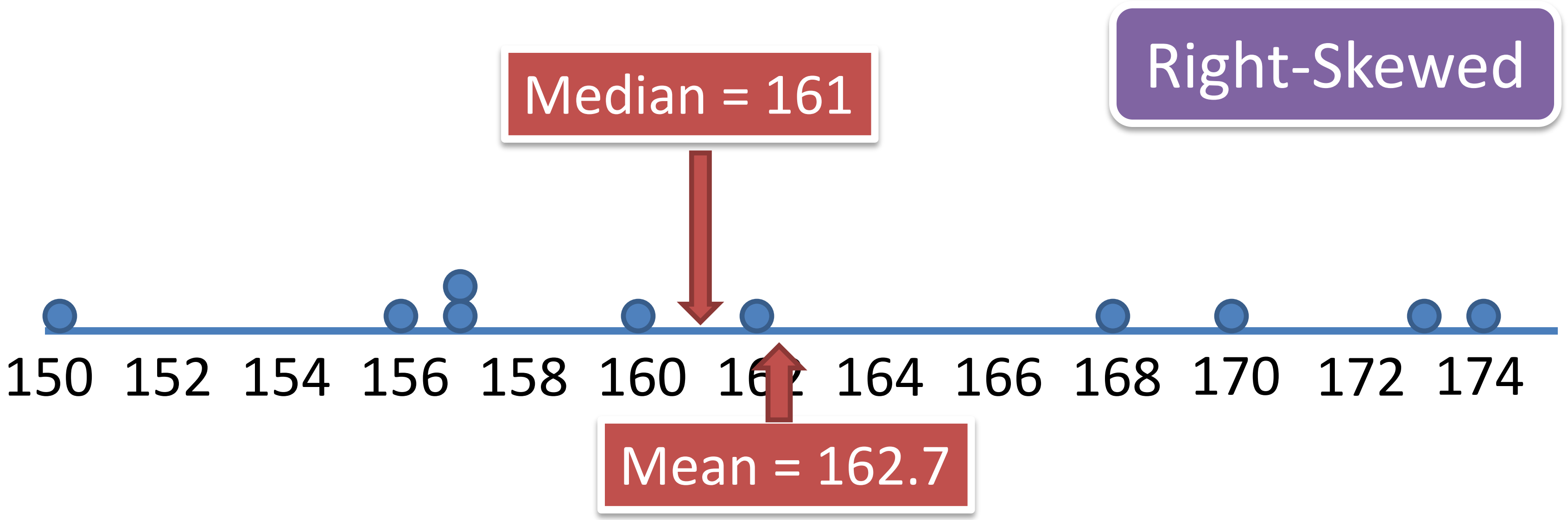
$$Z_X = \frac{X - \overline{X}}{SD}$$

- A data value is considered an extreme outlier if its Z-score is less than -3.0 or greater than +3.0.
- The larger the absolute value of the Z-score, the farther the data value is from the mean.

Shape

Let's see this height data again:

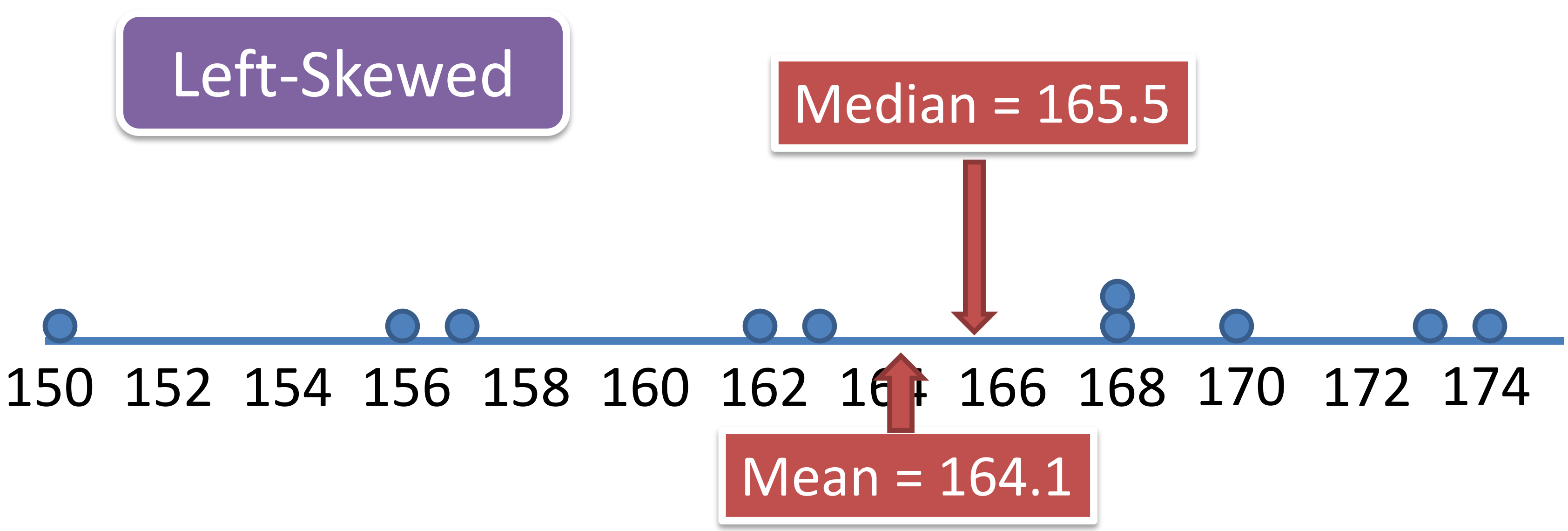
160 157 162 170 168 174 156 173 157 150



Shape

What if the height data is like this:

163 168 162 170 168 174 156 173 157 150

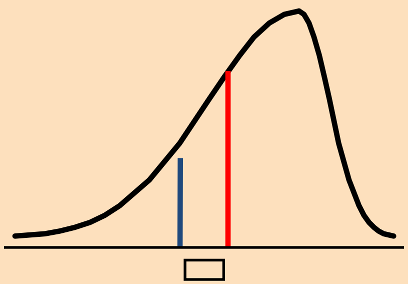


Shape

Describes how data are distributed

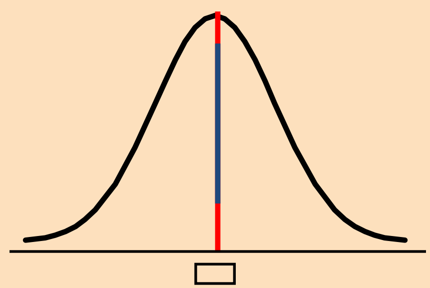
Left-Skewed

Mean < Median



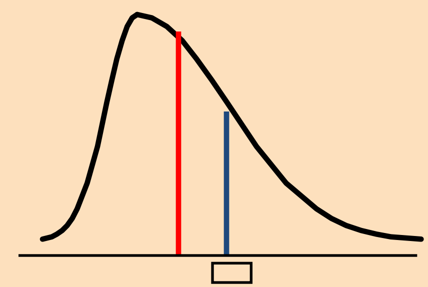
Symmetric

Mean = Median



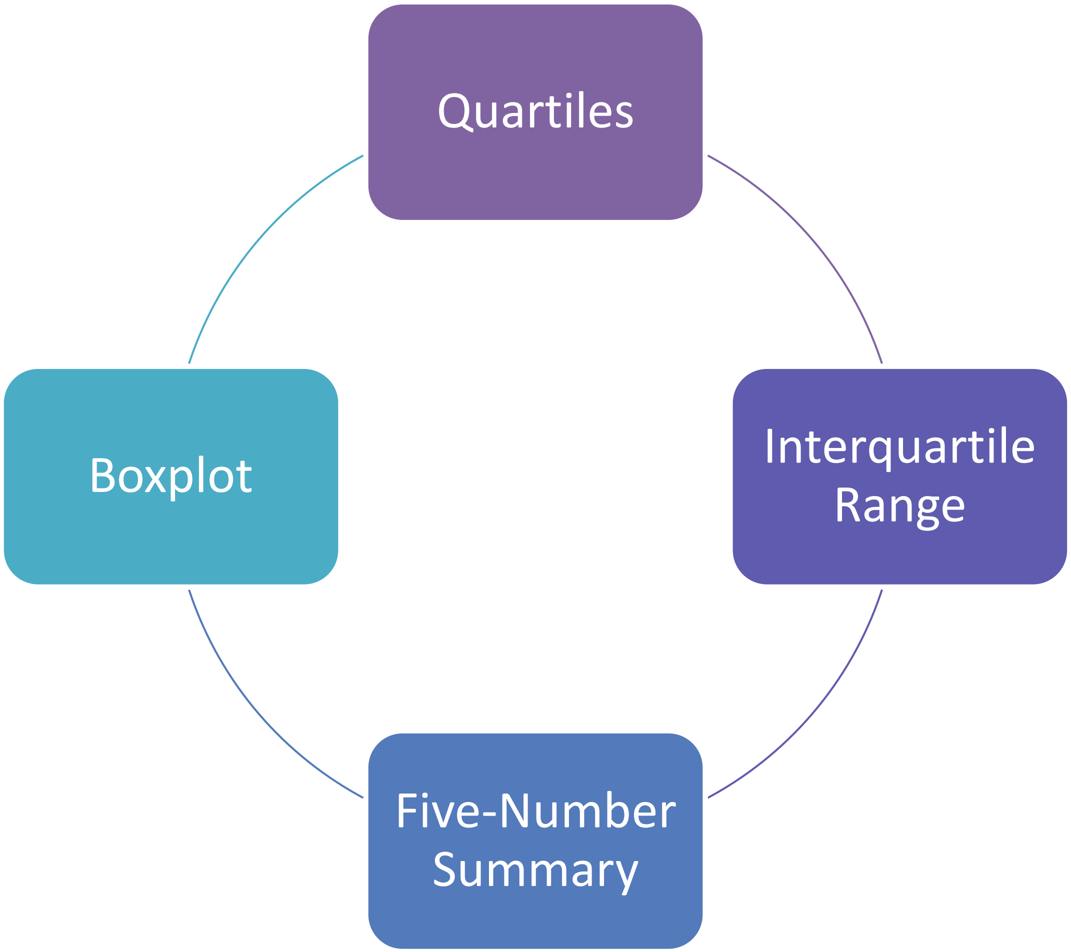
Right-Skewed

Median < Mean

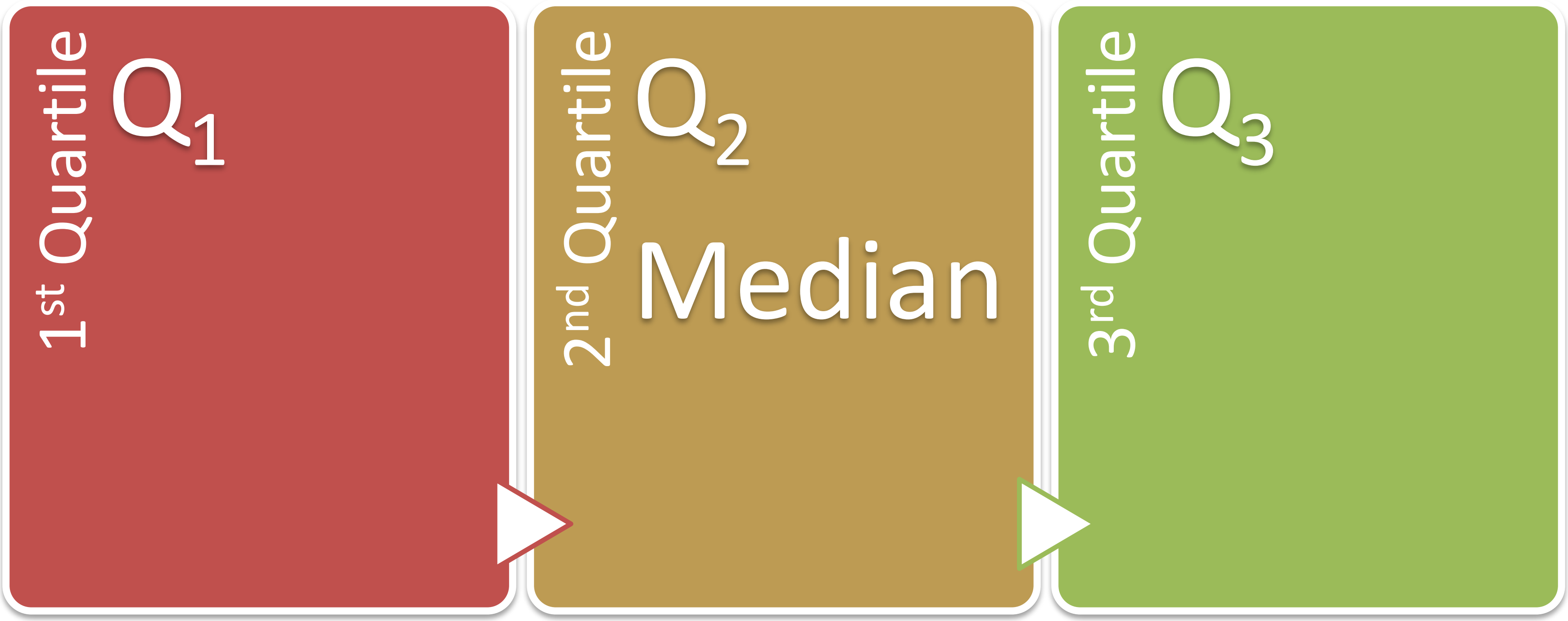


EXPLORING NUMERICAL DATA

Exploring Numerical Data



Quartiles



Quartiles

Let’s consider this height data:

160 157 162 170 168 174 156

What is the Q_1, Q_2 and Q_3?

$Q_1 = 157$

$Q_2 = 162$ (Median)

$Q_3 = 170$

Quartiles

Let’s then consider this height data:

160 157 162 170 168 174 156 173 150

What is the Q_1, Q_2 and Q_3?

$Q_1 = 156.5$

$Q_2 = 162$ (Median)

$Q_3 = 171.5$

Quartiles

And this height data:

160 157 162 170 168 174 156 173 157 150

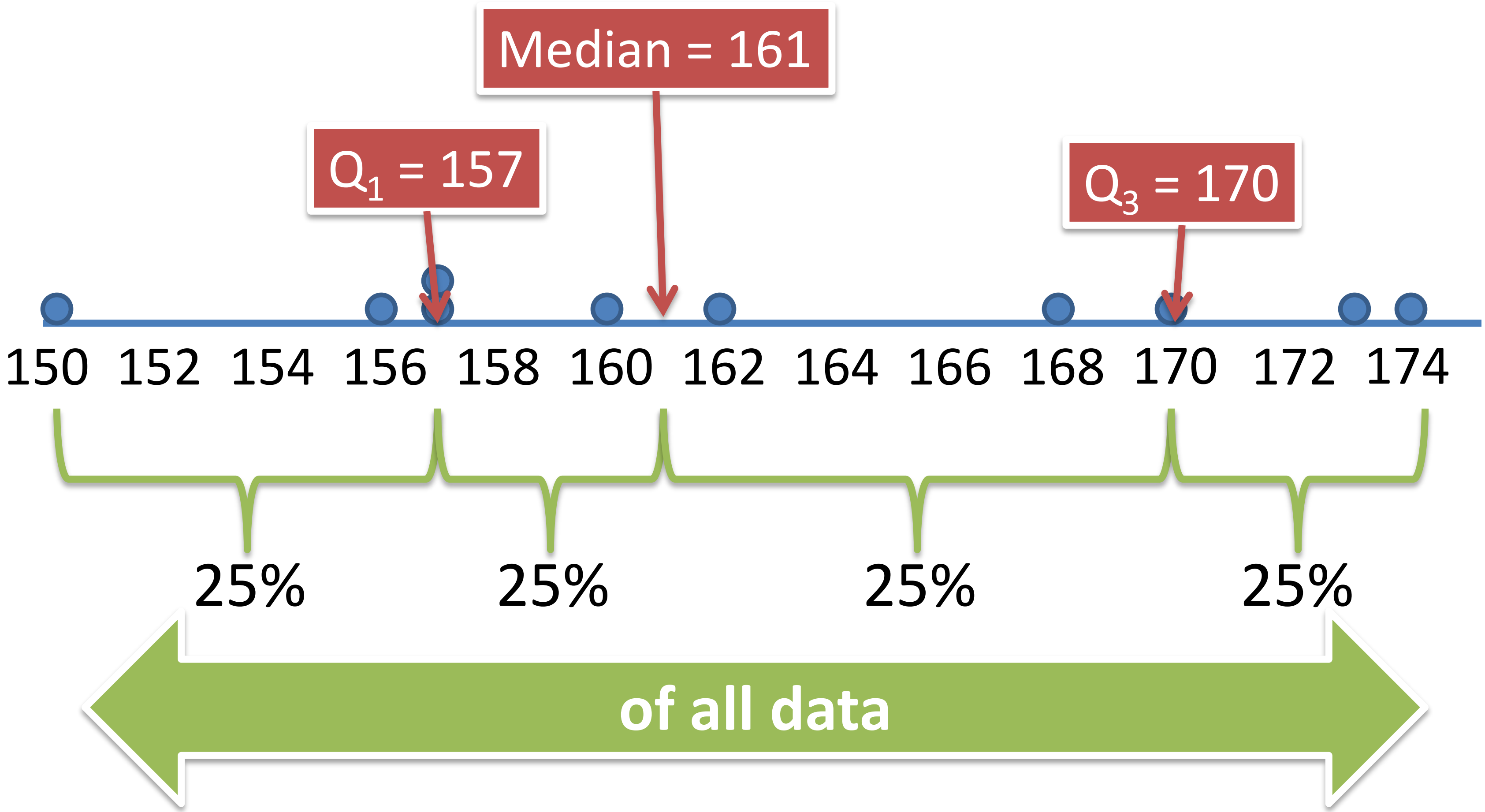
What is the Q_1, Q_2 and Q_3?

$Q_1 = 157$

$Q_2 = 161$ (Median)

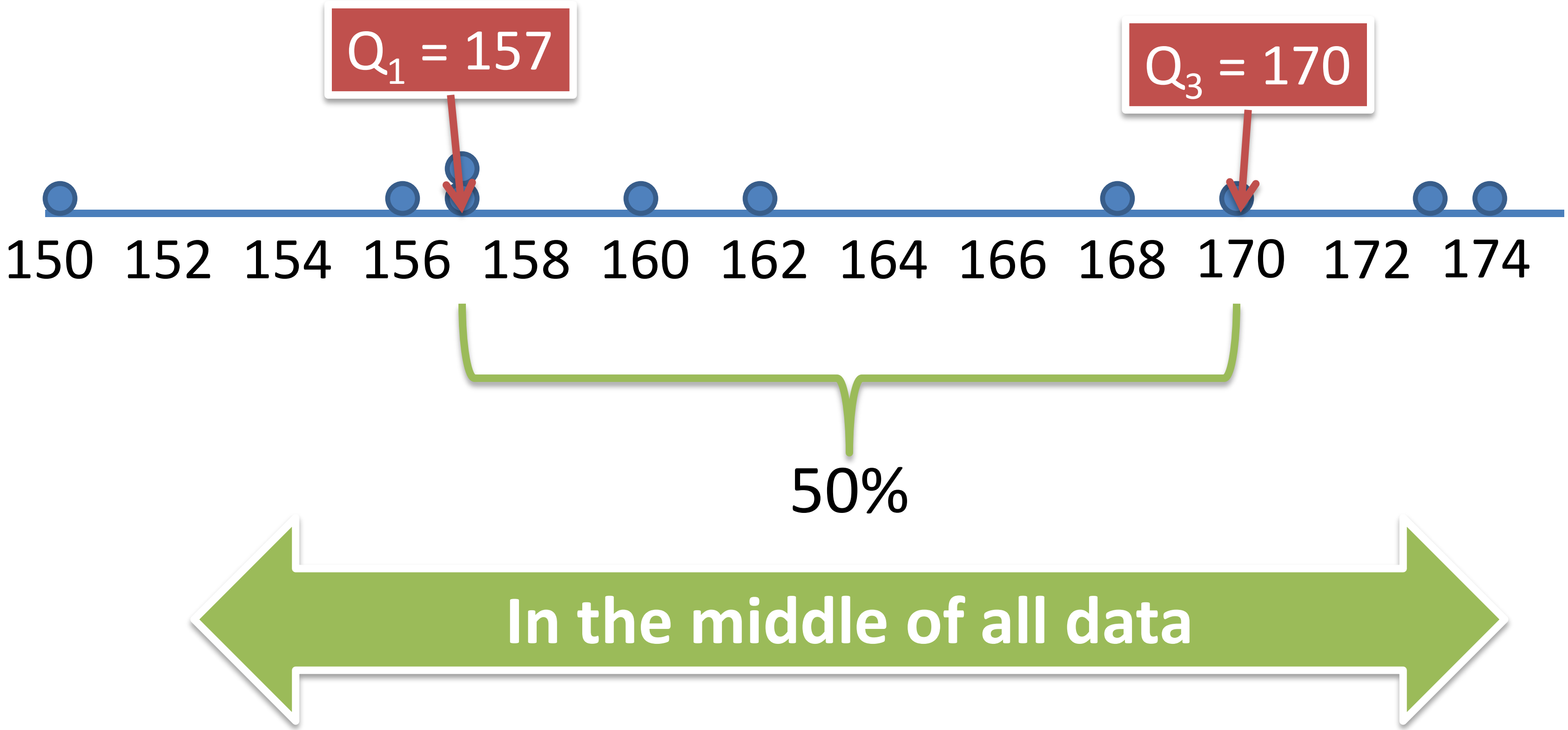
$Q_3 = 170$

Quartiles

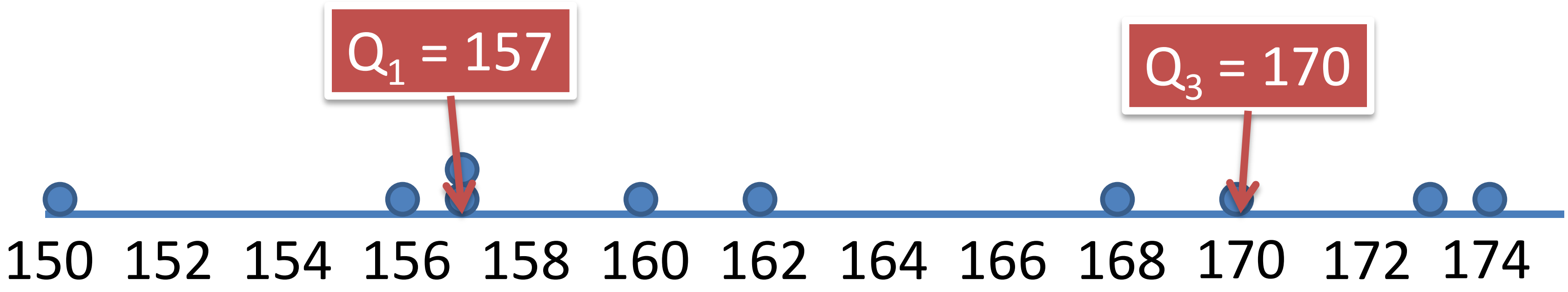


Median = 161
Q1 = 157
Q3 = 170
150 152 154 156 158 160 162 164 166 168 170 172 174
25%
25%
25%
25%
of all data

Interquartile Range



Interquartile Range



What is the Interquatile range?

Interquartile Range = 170 – 157 = 13

Interquartile Range

$$Interquartile\ Range = Q_3 - Q_1$$

Five-Number Summary

$$X_{\min} \quad Q_1 \quad Median \quad Q_3 \quad X_{\max}$$

Five-Number Summary

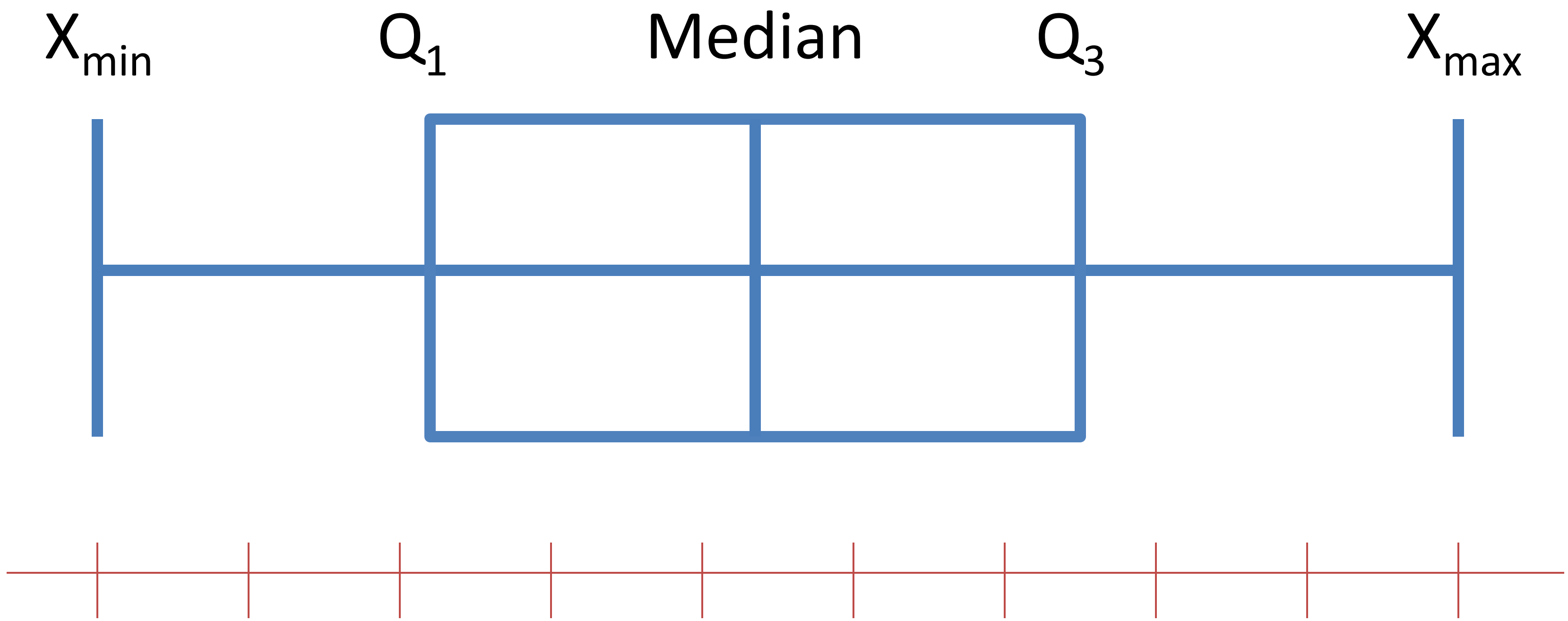
Let's see again this height data:

160 157 162 170 168 174 156 173 157 150

What is the Five-Number Summary?

150 157 161 170 174

Boxplot



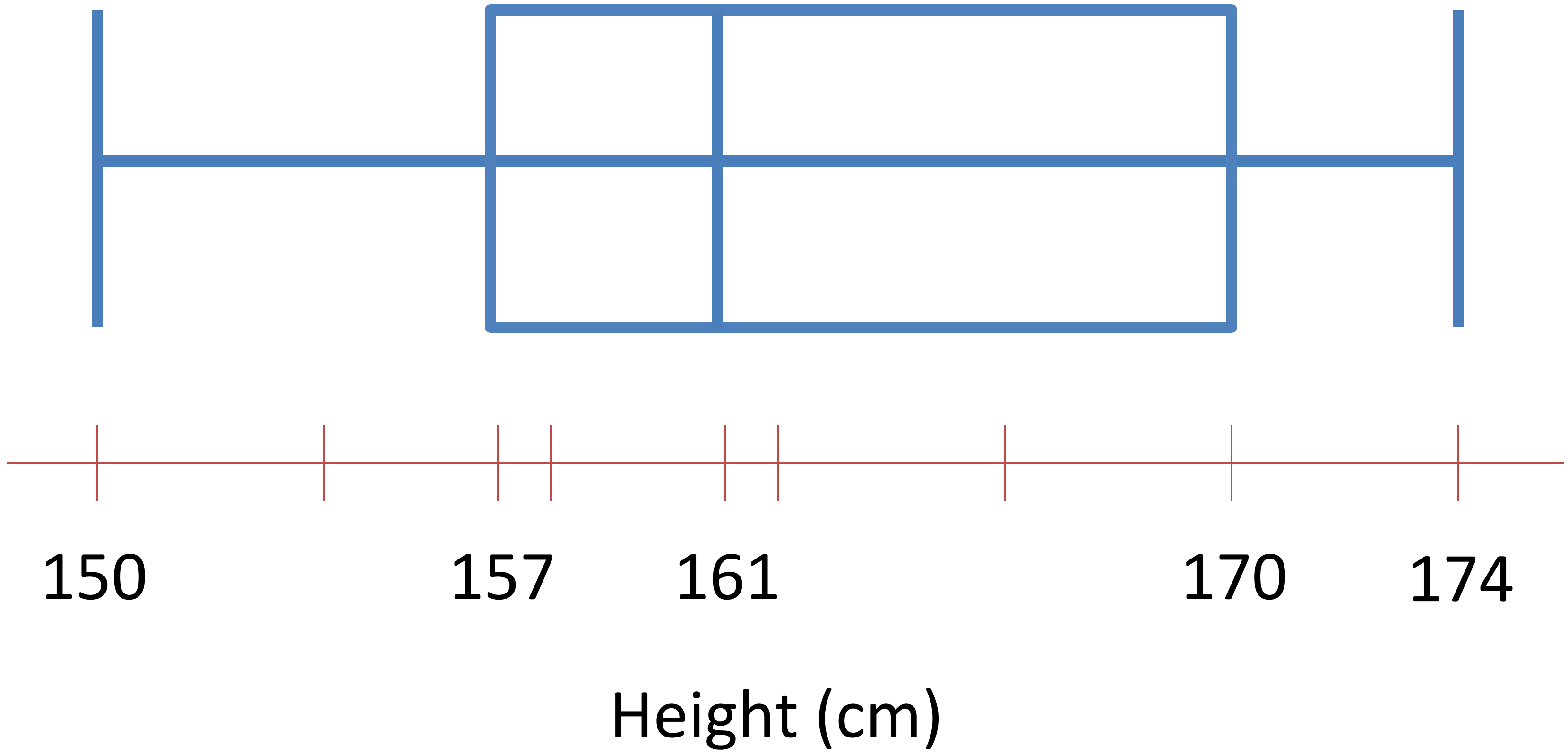
Boxplot

Let’s see again this height data:

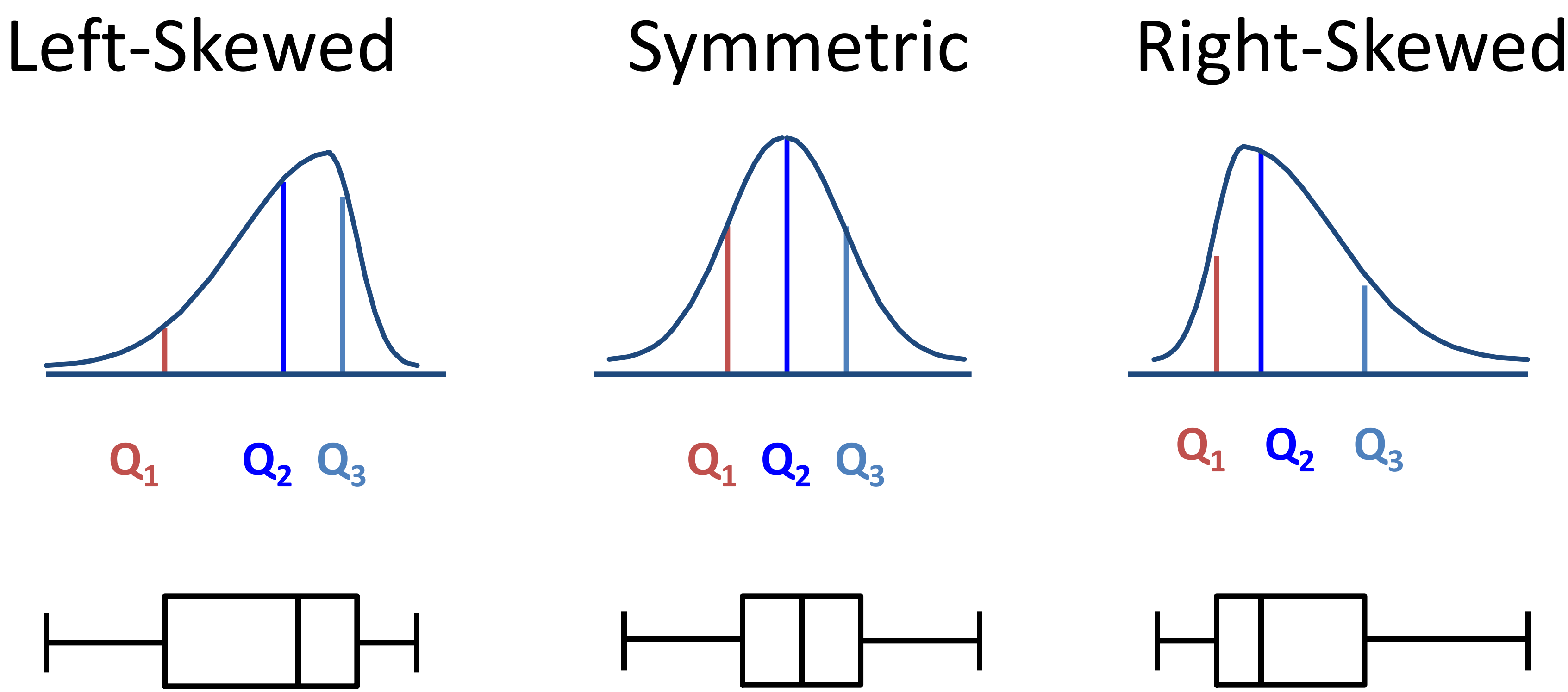
160 157 162 170 168 174 156 173 157 150

Construct the Boxplot?

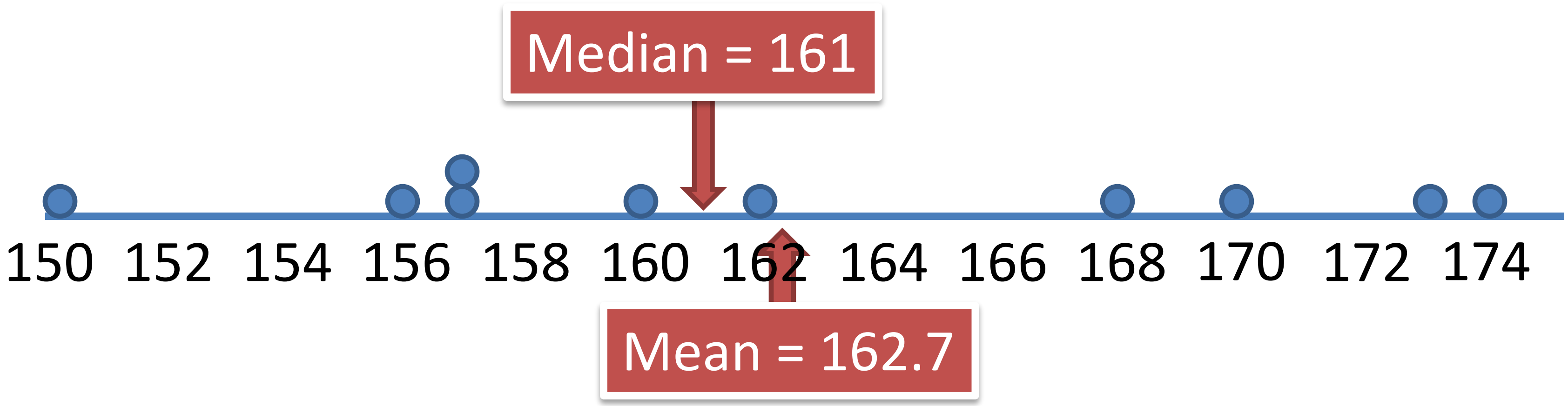
Boxplot for the Height of Business Statistic's Student 2014



Distribution Shape and The Boxplot



Karl Pearson's Measure of Skewness

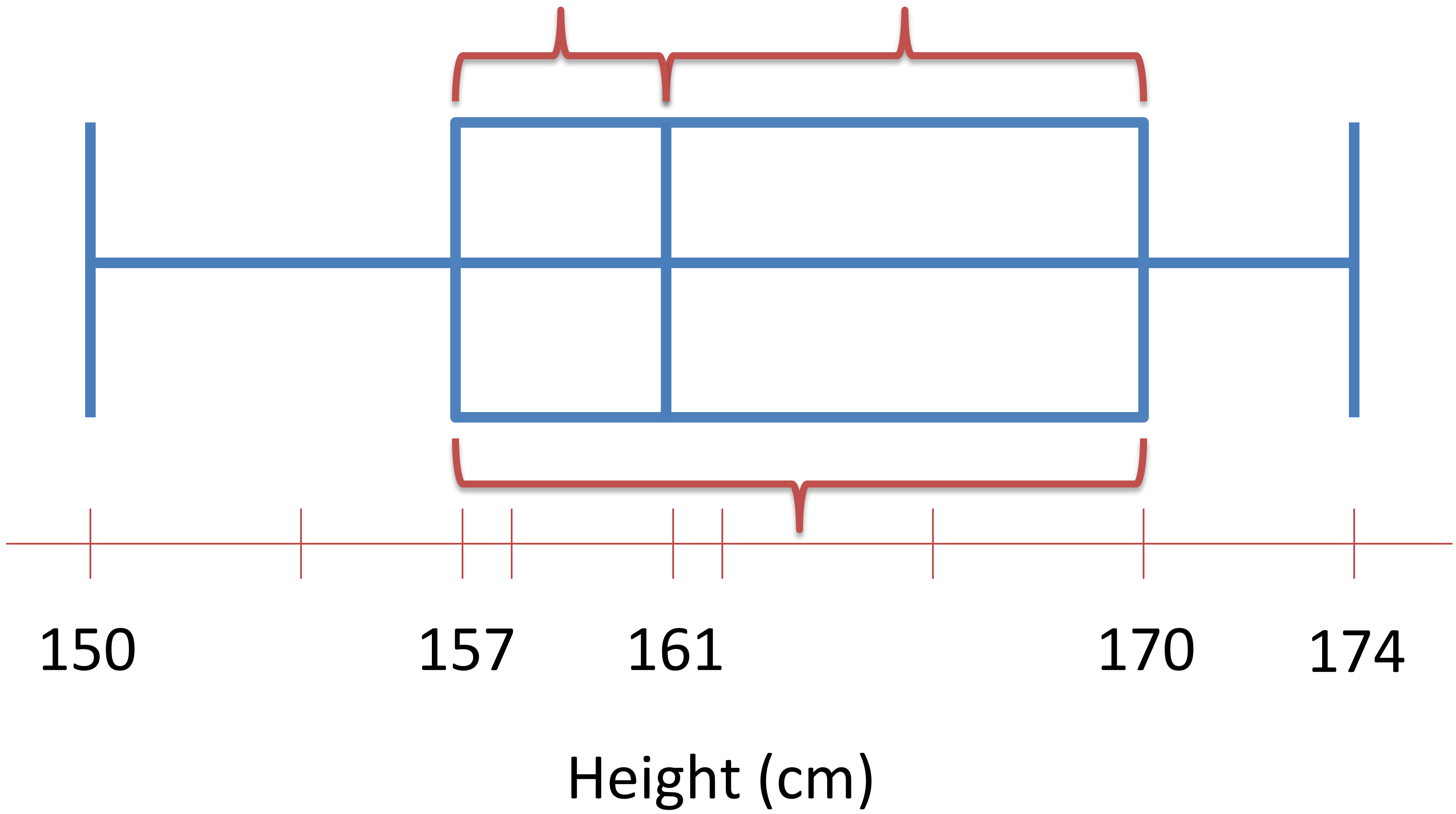


$$S_k = \frac{3(162.7 - 161)}{8.125} = 0.63$$

Karl Pearson's Measure of Skewness

$$S_k = \frac{3(\overline{X} - Median)}{S}$$

Bowley's Formula for Measuring Skewness



Bowley's Formula for Measuring Skewness

$$S_k = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_1)}$$

EXERCISE

3.10

This is the data of the amount that sample of nine customers spent for lunch ($) at a fast-food restaurant:

4.20 5.03 5.86 6.45 7.38 7.54 8.46 8.47 9.87

a. Compute the mean and median.
b. Compute the variance, standard deviation, and range
c. Are the data skewed? If so, how?
d. Based on the results of (a) through (c), what conclusions can you reach concerning the amount that customers spent for lunch?

3.62

In New York State, savings banks are permitted to sell a form of life insurance called savings bank life insurance (SBLI). The approval process consists of underwriting, which includes a review of the application, a medical information bureau check, possible requests for additional medical information and medical exams, and a policy compilation stage, during which the policy pages are generated and sent to the bank for delivery. The ability to deliver approved policies to customers in a timely manner is critical to the profitability of this service to the bank. During a period of one month, a random sample of 14 approved policies was selected, and the following were the total processing times

3.62

73 19 16 64 28 28 31 90 60 56 31 56 22 18

a. Compute the mean, median, first quartile, and third quartile.

b. Compute the range, interquartile range, variance, and standard deviation.

c. Are the data skewed? If so, how?

d. What would you tell a customer who enters the bank to purchase this type of insurance policy and asks how long the approval process takes?

3.22

In 2006-2009, the value of precious metals changed rapidly. The data in the following table represent the total rate of return (in percentage) for platinum, gold, an silver from 2006 through 2009:

Year	Platinum	Gold	Silver
2009	62.7	25.0	56.8
2008	-41.3	4.3	-26.9
2007	36.9	31.9	14.4
2006	15.9	23.2	46.1

3.22

a. Compute the geometric mean rate of return per year for platinum, gold, and silver from 2006 through 2009.

b. What conclusions can you reach concerning the geometric mean rates of return of the three precious metals?

3.66

The table contains data on the calories and total fat (in grams per serving) for a sample of 12 veggie burgers.

Calories	Fat
110	3.5
110	4.5
90	3.0
90	2.5
120	6.0
130	6.0
120	3.0
100	3.5
140	5.0
70	0.5
100	1.5
120	1.5

3.66

a. For each variable, compute the mean, median, first quartile, and third quartile.
b. For each variable, compute the range, variance, and standard deviation
c. Are the data skewed? If so, how?
d. Compute the coefficient of correlation between calories and total fat.
e. What conclusions can you reach concerning calories and total fat?

THANK YOU