Business Statistic

Week 3.1 - Index Number

Agenda

Time	Activity
60 minutes	Index Number
30 minutes	Exercise

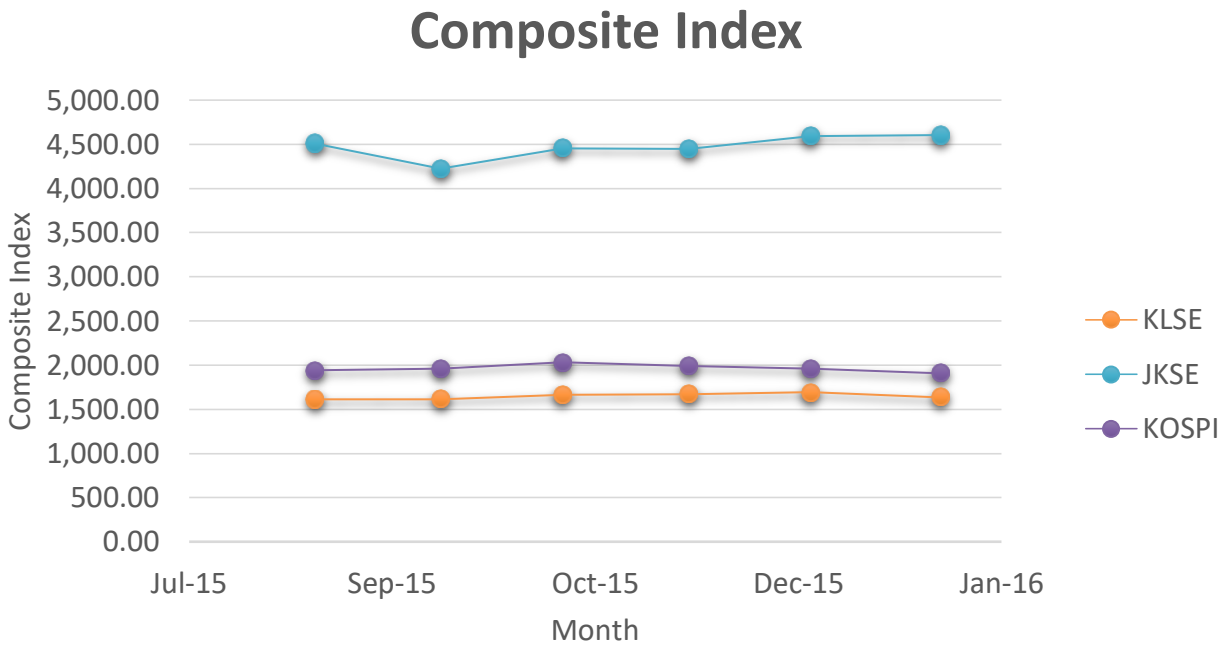
Objectives

At the end of this session the students will be able to

- Explain what an index number is
- Compile a simple index number
- Identify different types of index numbers
- Explain how index numbers can be used in practice

Introduction

The best-known index is the **consumer price index**, which gives a sort of "average" value for inflation based on price changes for a group of selected products. The **JKSE Composite Index** for the Jakarta Stock Exchanges, respectively, are also index numbers.



Index Number

- Simple Index Number
- Unweighted Aggregate Index
- Weighted Aggregate Index

SIMPLE INDEX NUMBER

Simple Index Number

INDEX NUMBER A number that expresses the relative change in **price**, **quantity**, or **value** compared to a base period.

Let's see this problem:

According to Statistics Canada, in 1995 the average salary of wage earners 15 years and older in Newfoundland and Labrador was $20,828 per year. In 2001, it was $24,165 per year. What is the index of yearly earnings of workers over age 15 in Newfoundland and Labrador for 2001 based on 1995?

Simple Index Number

INDEX NUMBER A number that expresses the relative change in **price**, **quantity**, or **value** compared to a base period.

Solution:

$$I = \frac{\textbf{\textit{Average yearly income of wage earners over 15 in 2001}}}{\textbf{\textit{Average yearly income of wage earners over 15 in 1995}}} \ (100)$$

$$I = \frac{24,165}{20,828} \ (100) = 116.0$$

Simple Index Number

Now, Let's consider this problem:

Statistics Canada results show that the number of farms in Canada dropped from 276,548 in 1996, to an estimated 246,923 in 2001. What is the index for the number of farms in 2001 based on the number in 1996?

Solution:

$$\boldsymbol{I = \frac{Number\ of\ farm\ in\ 2001}{Number\ of\ farm\ in\ 1996}\ (100)}$$

$$\boldsymbol{I = \frac{276.548}{246.923}\ (100) = 89,3}$$

Simple Index Number

Note from the previous discussion that:

1. Index numbers are actually percentages because they are based on the number 100. However, the percent symbol is usually omitted.
2. Each index number has a base period. The current base period for the Consumer Price Index is 2012 = 100, changed from 2007 = 100 in January 2014.
3. Most business and economic indexes are computed to the nearest whole number, such as 214 or 96, or to the nearest tenth of a percent, such as 83.4 or 118.7.

Why convert data to indexes?

An index is a convenient way to express a change in a diverse group of items.

Why convert data to indexes?

Converting data to indexes also makes it easier to assess the trend in a series composed of exceptionally large numbers.

Year	Indonesia's Official Reserve Assets Position
2013	99,387,000,000.00
2014	111,862,000,000.00
Difference	**12,475,000,000.00**

$$\frac{\textit{Official Reserve Asset Position in } 2014}{\textit{Official Reserve Asset Position in } 2013}$$

$$= \frac{111,862,000,000}{99,387,000,000} \times 100 = 113$$

Construction of Index Numbers

- Simple Price Index

$$PI = \frac{P_t}{P_0} \times 100$$

Where:

PI : Price Index

P_t : Price in the given period or selected period

P_0 : Price in the base period

Construction of Index Numbers

Let's take a look on the table below:

Prices and Price Index of a Benson Automatic Stapler

Year	Price of Stapler ($)	Price Index (1990=100)
1985	18	
1990	20	100
1991	22	
1992	23	
2001	38	

The base period used here is year 1990

Construction of Index Numbers

Let's take a look on the table below:

Prices and Price Index of a Benson Automatic Stapler

Year	Price of Stapler ($)	Price Index (1990=100)
1985	18	90
1990	20	100
1991		110
1992		115
2001		190

$$PI_{1985} = \frac{18}{20} \times 100$$

The base period used here is year 1990

Construction of Index Numbers

Simple Quantity Index

$$QI = \frac{Q_t}{Q_0} \times 100$$

Where:

QI : Quantity Index

Q_t : Quantity in the given period or selected period

Q_0 : Quantity in the base period

Simple Value Index

$$VI = \frac{(P_t \times Q_t)}{(P_0 \times Q_0)} \times 100$$

VI : Value Index

P_t : Price in the given period or selected period

P_0 : Price in the base period

Exercise

The following table contains the exchange rate of Indonesia Rupiahs to US dollars each year, convert this series into index numbers with the year 2011 as a base year.

Year	IDR per USD	Index (2011=100)
2011	9,098	
2012	9,788	
2013	12,180	
2014	12,545	
2015	13,830	

Answer

The following table contains the exchange rate of Indonesia Rupiahs to US dollars each year, convert this series into index numbers with the year 2011 as a base year.

Year	IDR per USD	Index (2011=100)
2011	9,098	100
2012	9,788	108
2013	12,180	134
2014	12,545	138
2015	13,830	152

UNWEIGHTED AGGREGATE INDEXES

Unweighted Indexes

Simple Average of the Price Indexes

Simple Aggregate Index

Simple Average of the Price Indexes

Table below reports the prices for several food items for the years 1995 and 2005. We would like to develop an index for this group of food items for 2005, using 1995 as the base.

Computation of Index for Food Price 2005, 1995=100

Item	1995 Price ($)	2005 Price ($)	Simple Index
Bread white (loaf)	0.77	1.98	
Eggs (dozen)	1.85	1.84	
Milk (litre) white	0.88	1.98	
Apples, red delicious (500 g)	1.46	1.75	
Orange juice (355 ml concentrate)	1.58	1.70	
Coffee, 100% ground roast (400 g)	4.40	3.99	
Total	**10.94**	**13.24**	

Simple Average of the Price Indexes

Table below reports the prices for several food items for the years 1995 and 2005. We would like to develop an index for this group of food items for 2005, using 1995 as the base.

Computation of Index for Food Price 2005, 1995=100

Item	1995 Price ($)	2005 Price ($)	Simple Index
Bread white (loaf)	0.77	1.98	257.1
Eggs (dozen)	1.85	1.84	99.5
Milk (litre) white	0.88	1.98	225.0
Apples, red delicious (500 g)	1.46	1.75	119.9
Orange juice (355 ml concentrate)	1.58	1.70	107.6
Coffee, 100% ground roast (400 g)	4.40	3.99	90.7
Total	**10.94**	**13.24**	**899.8**

Simple Average of the Price Indexes

SIMPLE AVERAGE OF THE PRICE INDEXES

$$PI = \frac{\sum PI_i}{n}$$

$$PI = \frac{899.8}{6} = 150$$

- This indicates that the mean of the group of indexes increased 50 percent from 1995 to 2005.

Simple Aggregate Index

Let's use the data from before:

Item	1995 Price ($)	2005 Price ($)
Bread white (loaf)	0.77	1.98
Eggs (dozen)	1.85	1.84
Milk (litre) white	0.88	1.98
Apples, red delicious (500 g)	1.46	1.75
Orange juice (355 ml concentrate)	1.58	1.70
Coffee, 100% ground roast (400 g)	4.40	3.99
Total	**10.94**	**13.24**

Simple Aggregate Price Index

$$API = \frac{\sum P_t}{\sum P_0} \times 100$$

$$PI = \frac{13.24}{10.94} \times 100 = 121$$

Simple Aggregate Index

Simple Aggregate Quantity Index

$$AQI = \frac{\sum Q_t}{\sum Q_0} \times 100$$

Where:

AQI : Aggregate Quantity Index

Q_t : Quantity in the given period or selected period

Q_0 : Quantity in the base period

Simple Aggregate Value Index

$$AVI = \frac{\sum (P_t \times Q_t)}{\sum (P_0 \times Q_0)} \times 100$$

AVI : Aggregate Value Index

P_t : Price in the given period or selected period

P_0 : Price in the base period

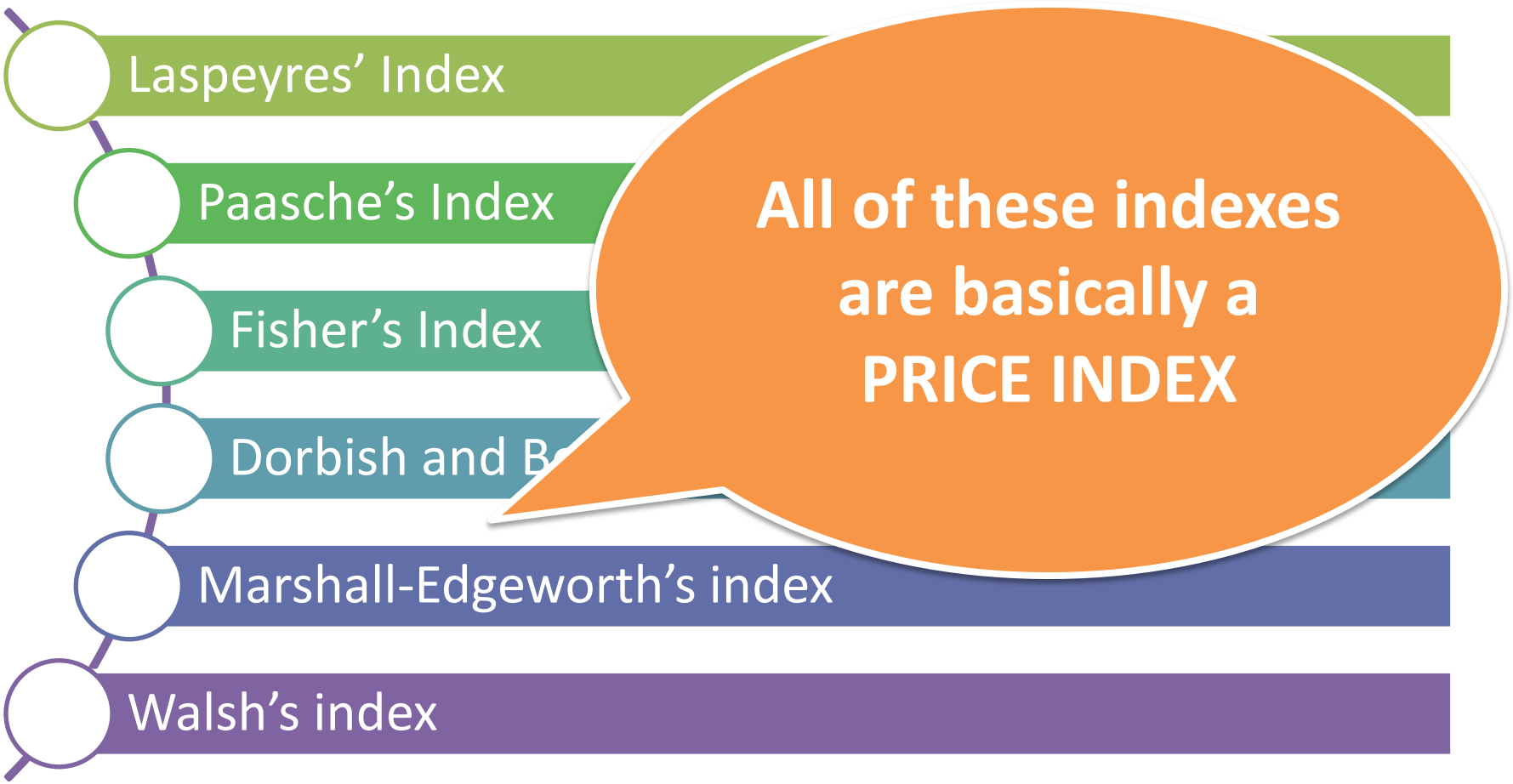
Simple Aggregate Index

- Because the value of a simple aggregate index can be influenced by the units of measurement, it is not used frequently.
- In our example the value of the index would differ significantly if we were to report the price of apples in tonnes rather than kilograms.
- Also, note the effect of coffee on the total index.
- For both the current year and the base year, the value of coffee is about 40 percent of the total index, so a change in the price of coffee will drive the index much more than any other item.
- So we need a way to appropriately “weight” the items according to their relative importance.

WEIGHTED AGGREGATE INDEXES

Weighted Indexes

Weighted Indexes



Laspeyres' Index

Etienne Laspeyres developed a method in the latter part of the 18th century to determine a weighted index using **base-period weights**.

Laspeyres' Price Index

$$LPI = \frac{\sum(P_t \times Q_0)}{\sum(P_0 \times Q_0)} \times 100$$

Where:

P_t : Price in the given period or selected period

P_0 : Price in the base period

Q_0 : Quantity in the base period

Laspeyres' Index

Table below contain the prices for the six food items and the number of units of each consumed by a typical family in 1995 and 2005

Item	1995		2005	
	Price ($)	Quantity	Price ($)	Quantity
Bread white (loaf)	0.77	50	1.98	55
Eggs (dozen)	1.85	26	2.98	20
Milk (litre) white	0.88	102	1.98	130
Apples, red delicious (500 g)	1.46	30	1.75	40
Orange juice, (355 ml concentrate)	1.58	40	1.7	41
Coffee, 100% ground roast (400 g)	4.4	12	4.75	12

Laspeyres' Index

Solution:

Item	P_0xQ_0	P_txQ_0
Bread white (loaf)	38.5	99
Eggs (dozen)	48.1	77.48
Milk (litre) white	89.76	201.96
Apples, red delicious (500 g)	43.8	52.5
Orange juice, (355 ml concentrate)	63.2	68
Coffee, 100% ground roast (400 g)	52.8	57
	336.16	**555.94**

$$\boldsymbol{LPI = \frac{555.94}{336.16} \times 100 = 165}$$

Paasche's Index

The Paasche index is an alternative. The procedure is similar, but instead of using base period weights, we use **current period weights**.

Paasche's Price Index

$$PPI = \frac{\sum(P_t \times Q_t)}{\sum(P_0 \times Q_t)} \times 100$$

Where:

P_t : Price in the given period or selected period

P_0 : Price in the base period

Q_t : Quantity in the given period or selected period

Paasche's Index

Table below contain the prices for the six food items and the number of units of each consumed by a typical family in 1995 and 2005

Item	1995		2005	
	Price ($)	Quantity	Price ($)	Quantity
Bread white (loaf)	0.77	50	1.98	55
Eggs (dozen)	1.85	26	2.98	20
Milk (litre) white	0.88	102	1.98	130
Apples, red delicious (500 g)	1.46	30	1.75	40
Orange juice, (355 ml concentrate)	1.58	40	1.7	41
Coffee, 100% ground roast (400 g)	4.4	12	4.75	12

Paasche's Index

Solution:

Item	P_0xQ_t	P_txQ_t
Bread white (loaf)	42.35	108.9
Eggs (dozen)	37	59.6
Milk (litre) white	114.4	257.4
Apples, red delicious (500 g)	58.4	70
Orange juice, (355 ml concentrate)	64.78	69.7
Coffee, 100% ground roast (400 g)	52.8	57
	369.73	**622.6**

$$\boldsymbol{PPI = \frac{622.6}{369.73} \times 100 = 168}$$

Laspeyres' Index vs. Paasche's Index

Laspeyres'

- Advantages:

  Requires quantity data from only the base period. This allows a more meaningful comparison over time. The changes in the index can be attributed to changes in the price.

- Disadvantages:

  Does not reflect changes in buying patterns over time. Also, it may overweight goods whose prices increase.

Paasche's

- Advantages:

  Because it uses quantities from the current period, it reflects current buying habits.

- Disadvantages:

  It requires quantity data for each year, which may be difficult to obtain. Because different quantities are used each year, it is impossible to attribute changes in the index to changes in price alone. It tends to overweight the goods whose prices have declined. It requires the prices to be recomputed each year.

Fisher's Index

In an attempt to offset Laspeyres' index and Paasche Index shortcomings, Irving Fisher, in his book *The Making of Index Numbers,* published in 1922, proposed an index called **Fisher's ideal index.** It is the **geometric mean** of the Laspeyres and Paasche indexes.

- **Fisher's Ideal Index**

$$FII = \sqrt{LPI \times PPI}$$

Where:

LPI : Laspeyres's Price Index

PPI : Paasche's Price Index

Fisher's Index

From the problem before, we got:

- LPI = 165
- PPI = 168

Therefore,

$$\boldsymbol{FII = \sqrt{165 \times 168} = 166}$$

Dorbish and Bowley's Index

Another attempt to consider taking an evenly weighted average of these fixed-basket price indices as a single estimator of price change between the two periods is developed by Drobisch (1871) and Bowley (1901).

- **Dorbish and Bowley's Index**

$$DBI = \frac{LPI + PPI}{2}$$

Where:

LPI : Laspeyres's Price Index

PPI : Paasche's Price Index

Dorbish and Bowley's Index

From the problem before, we got:

- LPI = 165
- PPI = 168

Therefore,

$$\boldsymbol{DBI = \frac{165 + 168}{2} = 166.5}$$

Marshall-Edgeworth's Index

Marshall (1887) and *Edgeworth* (1925) also try to develop *price index* formula to deal with problems related to laspeyres' index and paasche's index by using **arithmetic mean of the quantities**.

- **Marshall-Edgeworth's index**

$$\boldsymbol{LPI = \frac{\sum(P_t(Q_0 + Q_t))}{\sum(P_0(Q_0 + Q_t))} \times 100}$$

Where:

P_t : Price in the given period or selected period

P_0 : Price in the base period

Q_t : Quantity in the given period or selected period

Q_0 : Quantity in the base period

Marshall-Edgeworth's Index

Table below contain the prices for the six food items and the number of units of each consumed by a typical family in 1995 and 2005

Item	1995		2005	
	Price ($)	Quantity	Price ($)	Quantity
Bread white (loaf)	0.77	50	1.98	55
Eggs (dozen)	1.85	26	2.98	20
Milk (litre) white	0.88	102	1.98	130
Apples, red delicious (500 g)	1.46	30	1.75	40
Orange juice, (355 ml concentrate)	1.58	40	1.7	41
Coffee, 100% ground roast (400 g)	4.4	12	4.75	12

Marshall-Edgeworth's Index

Solution:

Item	Q_0+Q_t	$P_0(Q_0+Q_t)$	$P_t(Q_0+Q_t)$
Bread white (loaf)	105	80.85	207.9
Eggs (dozen)	46	85.1	137.08
Milk (litre) white	232	204.16	459.36
Apples, red delicious (500 g)	70	102.2	122.5
Orange juice, (355 ml concentrate)	81	127.98	137.7
Coffee, 100% ground roast (400 g)	24	105.6	114
		705.89	1178.54

$$\boldsymbol{MEI = \frac{1178.54}{705.89} \times 100 = 167}$$

Walsh's index

Correa Moylan Walsh (1901) also saw the price index number problem in the using fixed-basket index (laspeyres' and paasche index), and suggest using **geometric mean of both quantities**.

- **Walsh's index**

$$LPI = \frac{\sum(P_t\sqrt{Q_0 \times Q_t})}{\sum(P_0\sqrt{Q_0 \times Q_t})} \times 100$$

Where

P_t : [illegible] period

P_0 : Price in the base period

Q_t : Quantity in the given period or selected period

Q_0 : Quantity in the base period

Walsh's index

Table below contain the prices for the six food items and the number of units of each consumed by a typical family in 1995 and 2005

Item	1995		2005	
	Price ($)	Quantity	Price ($)	Quantity
Bread white (loaf)	0.77	50	1.98	55
Eggs (dozen)	1.85	26	2.98	20
Milk (litre) white	0.88	102	1.98	130
Apples, red delicious (500 g)	1.46	30	1.75	40
Orange juice, (355 ml concentrate)	1.58	40	1.7	41
Coffee, 100% ground roast (400 g)	4.4	12	4.75	12

Walsh's index

Solution:

Item	$\sqrt{Q_0 x Q_t}$	$P_0(\sqrt{Q_0 x Q_t})$	$P_t(\sqrt{Q_0 x Q_t})$
Bread white (loaf)	52.44	40.38	103.83
Eggs (dozen)	22.80	42.19	67.95
Milk (litre) white	115.15	101.33	228.00
Apples, red delicious (500 g)	34.64	50.58	60.62
Orange juice, (355 ml concentrate)	40.50	63.99	68.84
Coffee, 100% ground roast (400 g)	12.00	52.8	57
		351.26	586.25

$$\boldsymbol{WI = \frac{586.25}{351.26} \times 100 = 167}$$

Special-Purpose Indexes

- The Consumer Price Index (CPI)
- JKSE Composite Index
- NASDAQ Composite Index
- Wholesale Price Index
- Human Development Index

EXERCISE

Betts Electronics

Betts Electronics purchases three replacement parts for robotic machines used in their manufacturing process. Information on the price of the replacement parts and the quantity purchased is given below.

Part	Price ($)		Quantity	
	1999	2005	1999	2005
RC-33	0.5	0.6	320	340
SM-14	1.2	0.9	110	130
WC50	0.85	1	230	250

Betts Electronics

a. Compute a simple price index for each of the three items. Use 1999 as the base period.
b. Compute a simple aggregate price index for 2005. Use 1999 as the base period.
c. Compute Laspeyres' price index for 2005 using 1999 as the base period.
d. Compute Paasche's index for 2005 using 1999 as the base period.
e. Determine Fisher's ideal index using the values for the Laspeyres and Paasche indexes computed in the two previous problems.
f. Determine a value index for 2005 using 1999 as the base period.

THANK YOU