Business Statistic

Week 3

Time-Series Forecasting

Component Factors of Time-Series Models

Trend

- Overall long-term upward or downward movement in a time series

Cyclical Effect

- The up-and-down swings or movements through the series
- Cyclical movements vary in length, usually lasting from 2 to 10 years

Irregular or Random Effect

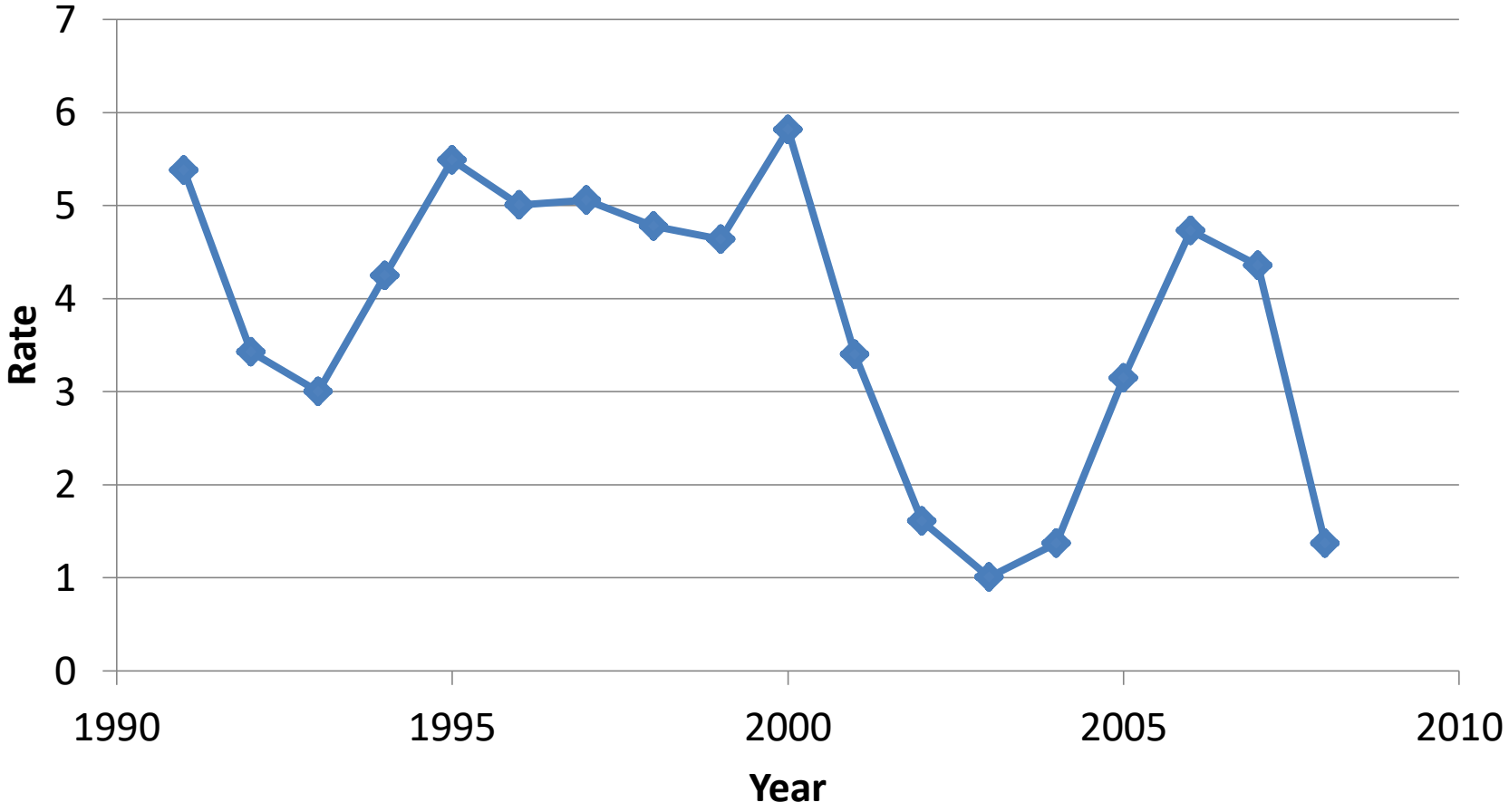
Seasonal Effect

- Monthly or quarterly cycle

Three-Month U.S. Treasury Bill Rate, 1991 through 2008

Year	Rate	Year	Rate	Year	Rate
1991	5.38	1997	5.06	2003	1.01
1992	3.43	1998	4.78	2004	1.37
1993	3.00	1999	4.64	2005	3.15
1994	4.25	2000	5.82	2006	4.73
1995	5.49	2001	3.40	2007	4.36
1996	5.01	2002	1.61	2008	1.37

Plot of Three-Month U.S. Treasury Bill Rate, 1991 through 2008



Moving Averages Smoothing (Rata-rata bergerak)

- To get better overall impression of the pattern of movement in the data over time
- Moving averages for a chosen period of length *L* consist of a series of means, each computed over time for a sequence of *L* observed values
- Example:
  - 3-year moving average → MA(3)
  - 5-month moving average → MA (5)

Moving Average Smoothing

Year	Actual	MA(3)	Rate	Actual	MA(3)
1991	5.38		2000	5.82	
1992	3.43				
1993	3.00				
1994	4.25				
1995	5.49		2004	1.37	
1996	5.01		2005	3.15	
1997	5.06		2006	4.73	
1998	4.78		2007	4.36	
1999	4.64		2008	1.37	

MA(3) = (5.38+3.43+3.00)/3
MA(3) = 3.94

Moving Average Smoothing

Year	Actual	MA(3)	Rate	Actual	MA(3)
1991	5.38		2000	5.82	
1992	3.43	3.94	2001	3.40	
1993	3.00		2002	1.61	
1994	4.25		2003	1.01	
1995	5.49		2004	1.37	
1996	5.01		2005	3.15	
1997	5.06		2006	4.73	
1998	4.78		2007	4.36	
1999	4.64		2008	1.37	

Moving Average Smoothing

Year	Actual	MA(3)	Rate	Actual	MA(3)
1991	5.38		2000	5.82	
1992	3.43	3.94	2001	3.40	
1993	3.00	3.56	2002	1.61	
1994	4.25		2003	1.01	
1995	5.49		2004	1.37	
1996	5.01		2005	3.15	
1997	5.06		2006	4.73	
1998	4.78		2007	4.36	
1999	4.64		2008	1.37	

Moving Average Smoothing

Year	Actual	MA(3)	Rate	Actual	MA(3)
1991	5.38		2000	5.82	4.62
1992	3.43	3.94	2001	3.40	3.61
1993	3.00	3.56	2002	1.61	2.01
1994	4.25	4.25	2003	1.01	1.33
1995	5.49	4.92	2004	1.37	1.84
1996	5.01	5.19	2005	3.15	3.08
1997	5.06	4.95	2006	4.73	4.08
1998	4.78	4.83	2007	4.36	3.49
1999	4.64	5.08	2008	1.37	

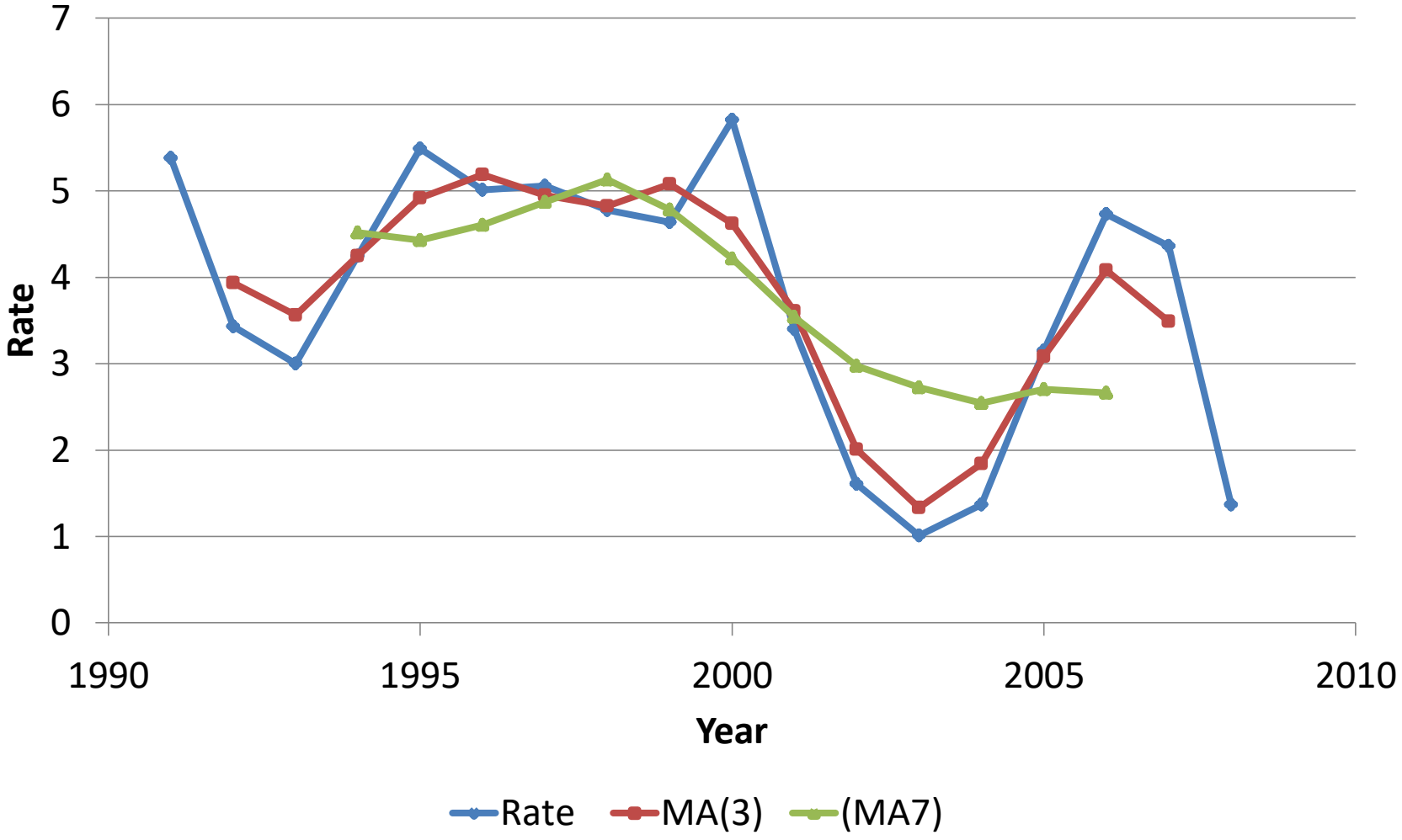
Exercise

- Compute the seven-year moving averages for this annual time series.

Exercise (Answer)

Year	Actual	MA(7)	Rate	Actual	MA(7)
1991	5.38		2000	5.82	4.22
1992	3.43		2001	3.40	3.54
1993	3.00		2002	1.61	2.98
1994	4.25	4.52	2003	1.01	2.73
1995	5.49	4.43	2004	1.37	2.55
1996	5.01	4.60	2005	3.15	2.71
1997	5.06	4.87	2006	4.73	2.67
1998	4.78	5.13	2007	4.36	
1999	4.64	4.79	2008	1.37	

Plot of Three-Month U.S. Treasury Bill Rate, 1991 through 2008



Moving Average Smoothing

- If cyclical fluctuations are present in the data, choose ***L*** that corresponds to the **estimated length of a cycle** in the series.
- For annual time-series data that has no obvious cyclical fluctuations, most people choose **three** years, **five** years, or **seven** years.
- Selecting moving averages that are longer than seven years is usually undesirable, because too many moving average values are missing.

Linear Trend Model

- Trend linear model is used when a straight-line trend adequately fits the data.
- Equation:

$$\hat{Y}_i = b_0 + b_1 X_i$$

where

$\hat{Y}_i$: predicted value of Y for observation i

X_i : value of X for observation i

b_0 : sample Y intercept

b_1 : sample slope

Linear Trend Model

$$\hat{Y}_i = b_0 + b_1 X_i$$

$$b_1 = \frac{\sum_{i=1}^{n} X_i Y_i - \frac{\left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right)}{n}}{\sum_{i=1}^{n} X_i^{\,2} - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}} \qquad b_0 = \bar{Y} - b_1 \bar{X}$$

Example

Revenues (in Billions of Dollars) for the Coca-Cola Company

Year	Revenue	Year	Revenue
1995	18.0	2003	21.0
1996	18.5	2004	21.9
1997	18.9	2005	23.1
1998	18.8	2006	24.1
1999	19.8	2007	28.9
2000	20.5	2008	31.9
2001	20.1	2009	31.0
2002	19.6		

Example (Answer)

Year	Revenue (Y_i)	X_i	$X_i.Y_i$	$(X_i)^2$
1995	18.0	0	0	0
1996	18.5	1	18.5	1
1997	18.9	2	37.8	4
1998	18.8	3	56.4	9
1999	19.8	4	79.2	16
2000	20.5	5	102.5	25
2001	20.1	6	120.6	36
2002	19.6	7	137.2	49
2003	21.0	8	168	64
2004	21.9	9	197.1	81
2005	23.1	10	231	100
2006	24.1	11	265.1	121
2007	28.9	12	346.8	144
2008	31.9	13	414.7	169
2009	31.0	14	434	196
Total	**336.1**	**105**	**2608.9**	**1015**

Example (Answer)

Year	Revenue (Y_i)	X_i	$X_i.Y_i$	$(X_i)^2$
1995	18.0	0	0	0
1996	18.5	1	18.5	1
1997	18.9	2	37.8	4
1998	18.8	3	56.4	9
1999	19.8	4	79.2	16
2000	20.5	5	102.5	25
[illegible]	20.1	6	12[illegible]	
[illegible]	19.6	7	13[illegible]	
[illegible]	21.0	8	16[illegible]	
2004	21.9	9	197.1	81
2005	23.1	10	231	100
2006	24.1	11	265.1	121
2007	28.9	12	346.8	144
2008	31.9	13	414.7	169
2009	31.0	14	434	196
Total	**336.1**	**105**	**2608.9**	**1015**

$\overline{Y} = 22.41$

$\overline{X} = 7$

Example (Answer)

$$b_1 = \frac{\sum_{i=1}^{n} X_i Y_i - \frac{\left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right)}{n}}{\sum_{i=1}^{n} X_i^{\,2} - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}}$$

Example (Answer)

$$b_1 = \frac{2608.9 - \frac{(105)(336.1)}{15}}{1015 - \frac{(105)^2}{15}}$$

$$b_1 = \frac{2608.9 - 2352.7}{1015 - 735} = \frac{256.2}{280} = 0.915$$

Example (Answer)

$$b_0 = \bar{Y} - b_1 \bar{X}$$

$$b_0 = 22.41 - 0.915 \times 7$$

$$b_0 = 22.41 - 6.405 = 16.002$$

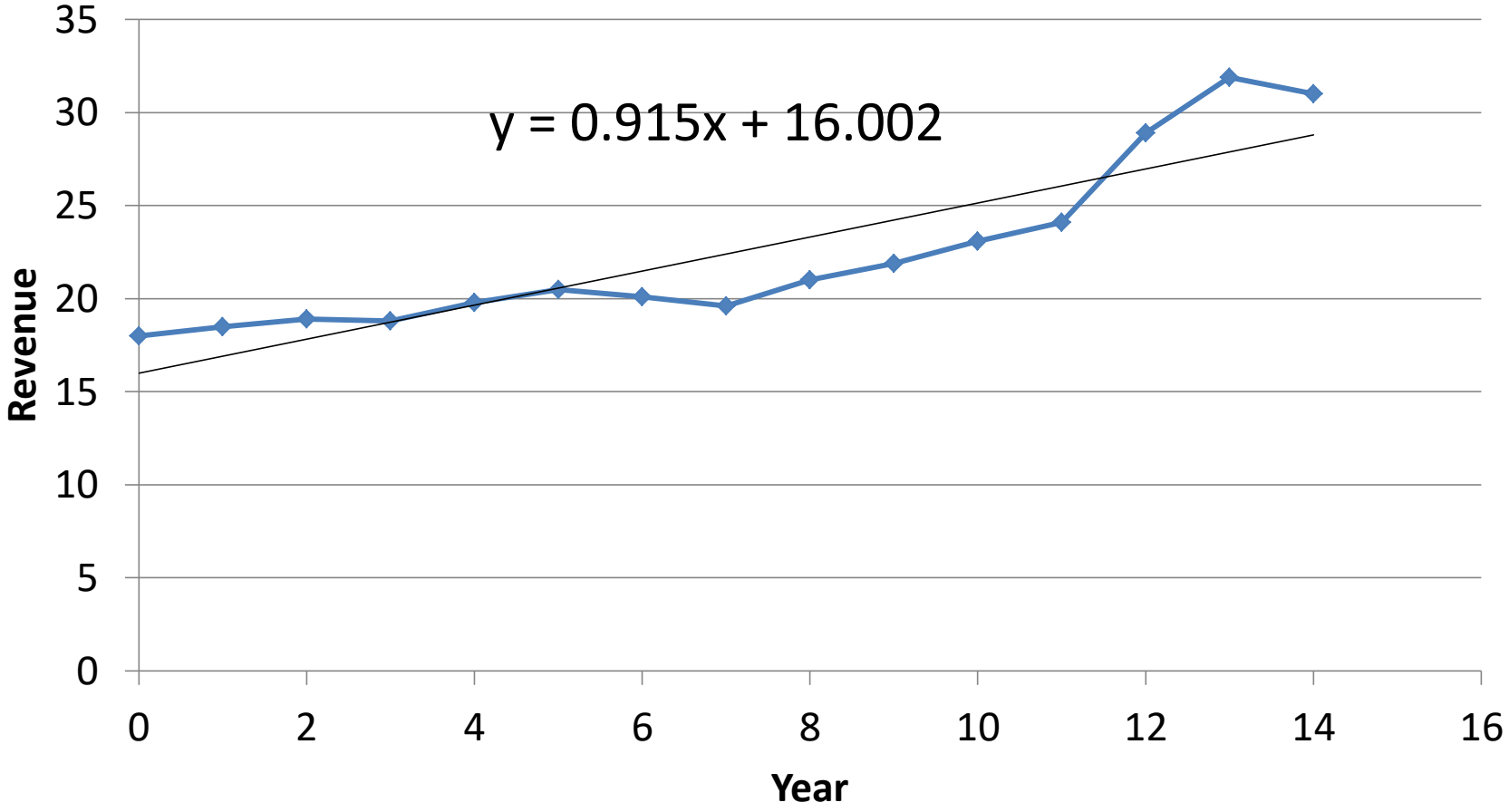
Example (Answer)

$$b_1 = 0.915 \qquad b_0 = 16.002$$

$$\hat{Y}_i = b_0 + b_1 X_i$$

$$\hat{Y}_i = 16.002 + 0.915 X_i$$

Plot of Revenues (in Billions of Dollars) for the Coca-Cola Company



EXERCISE

16.5

The following data provide the number of accidents in the NASCAR Sprint Cup series from 2001 to 2009

a. Fit a three-year and five-year moving average to the data

b. What conclusions can you reach concerning accidents in the NASCAR Sprint Cup series from 2001 to 2009?

Year	Accidents
2001	200
2002	186
2003	235
2004	204
2005	253
2006	237
2007	240
2008	211
2009	195

16.16

The data shown in the following table represent the yearly amount of solar power installed (in megawatts) in the United States from 2000 through 2008:

Year	Amount of Solar Power Installed
2000	18
2001	27
2002	44
2003	68
2004	83
2005	100
2006	140
2007	210
2008	250

16.16

a. Compute a linear trend forecasting equation.

b. Using the models in (a) what are your annual trend forecasts of the yearly amount of solar power installed (in megawatts) in the United States in 2009 and 2010?

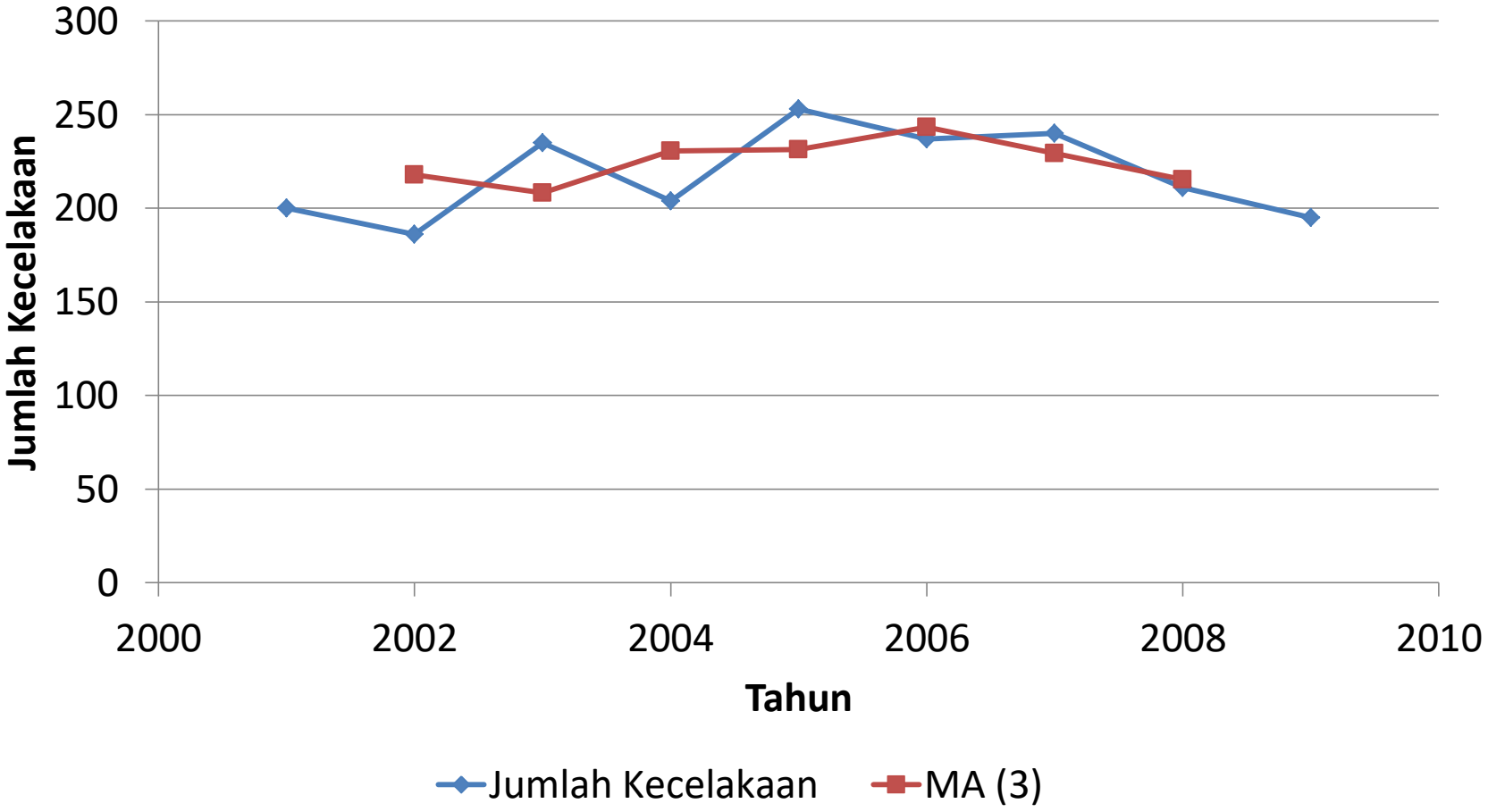
Year	Amount of Solar Power Installed
2000	18
2001	27
2002	44
2003	68
2004	83
2005	100
2006	140
2007	210
2008	250

ANSWER

16.5

Year	Accidents	MA (3)
2001	200	
2002	186	207
2003	235	208
2004	204	231
2005	253	231
2006	237	243
2007	240	229
2008	211	215
2009	195	

16.5


300
250
200
150
100
50
0
Jumlah Kecelakaan
2000
2002
2004
2006
2008
2010
Tahun
Jumlah Kecelakaan
MA (3)

16.16

Year	Amount of Solar Power Installed (Y_i)	X_i	X_iY_i	$(X_i)^2$
2000	18	0	0	0
2001	27	1	27	1
2002	44	2	88	4
2003	68	3	204	9
2004	83	4	332	16
2005	100	5	500	25
2006	140	6	840	36
2007	210	7	1470	49
2008	250	8	2000	64
Total	**940**	**36**	**5461**	**204**
Average	**104.44**	**4.00**		

16.16

$$b_1 = \frac{5461 - \frac{36.940}{9}}{204 - \frac{36^2}{9}}$$

$$b_1 = \frac{5461 - 3760}{204 - 144} = \frac{1701}{60} = 28.35$$

16.16

$$b_0 = \bar{Y} - b_1 \bar{X}$$

$$b_0 = 104.44 - 28.35 \times 4$$

$$b_0 = 104.44 - 113.4 = -8.9556$$

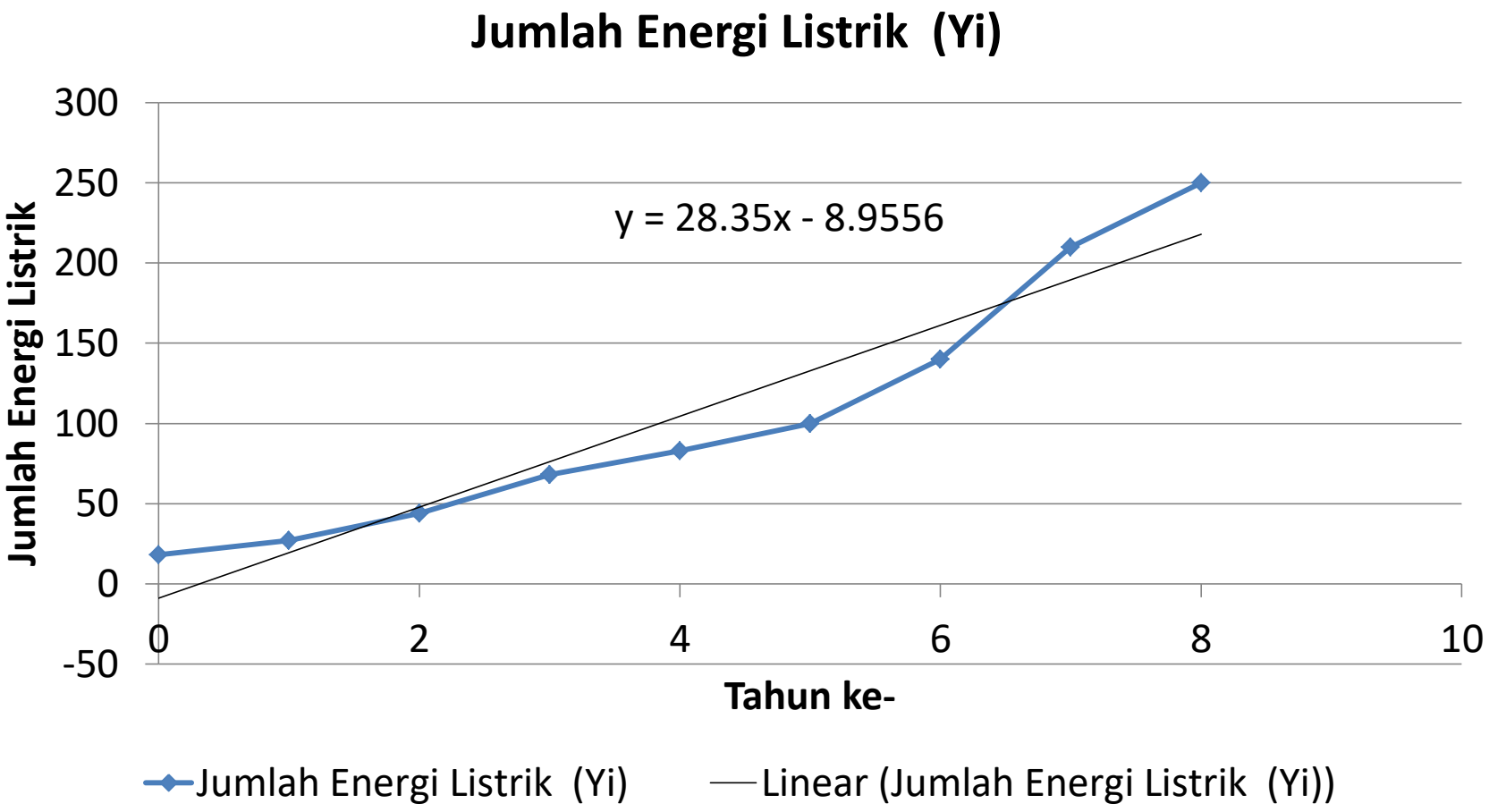
16.16

$b_1 = 28.35$ $b_0 = -8.9556$

$$\hat{Y}_i = b_0 + b_1 X_i$$

$$\hat{Y}_i = 28.35 X_i - 8.9556$$

16.16


Jumlah Energi Listrik (Yi)
y = 28.35x - 8.9556
Jumlah Energi Listrik
300
250
200
150
100
50
0
-50
0
2
4
6
8
10
Tahun ke-
Jumlah Energi Listrik (Yi)
Linear (Jumlah Energi Listrik (Yi))

THANK YOU