

Statistik Bisnis

Week 3.2

Time-Series Forecasting

Component Factors of Time-Series Models

Trend

- Overall long-term upward or downward movement in a time series

Cyclical Effect

- The up-and-down swings or movements through the series
- Cyclical movements vary in length, usually lasting from 2 to 10 years

Irregular or Random Effect

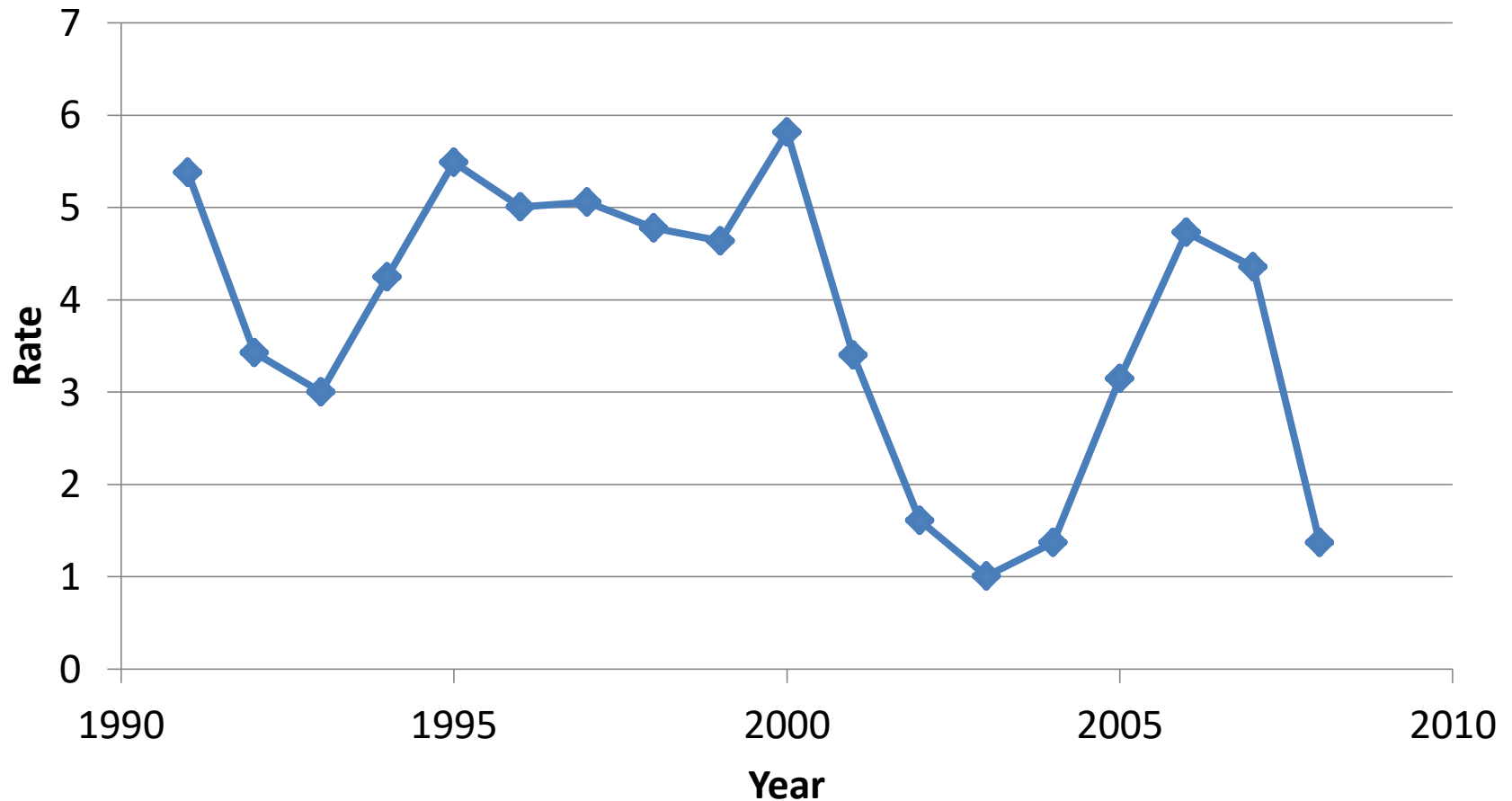
Seasonal Effect

- Monthly or quarterly cycle

Three-Month U.S. Treasury Bill Rate, 1991 through 2008

Year	Rate	Year	Rate	Year	Rate
1991	5.38	1997	5.06	2003	1.01
1992	3.43	1998	4.78	2004	1.37
1993	3.00	1999	4.64	2005	3.15
1994	4.25	2000	5.82	2006	4.73
1995	5.49	2001	3.40	2007	4.36
1996	5.01	2002	1.61	2008	1.37

Plot of Three-Month U.S. Treasury Bill Rate, 1991 through 2008



Moving Averages Smoothing (Rata-rata bergerak)

- To get better overall impression of the pattern of movement in the data over time
- Moving averages for a chosen period of length L consist of a series of means, each computed over time for a sequence of L observed values
- Example:
 - 3-year moving average \rightarrow MA(3)
 - 5-month moving average \rightarrow MA (5)

Moving Average Smoothing

Year	Actual	MA(3)	Rate	Actual	MA(3)
1991	5.38		2000	5.82	
1992	3.43				
1993	3.00				
1994	4.25		2003	1.01	
1995	5.49		2004	1.37	
1996	5.01		2005	3.15	
1997	5.06		2006	4.73	
1998	4.78		2007	4.36	
1999	4.64		2008	1.37	

$MA(3) = (5.38 + 3.43 + 3.00) / 3$
 $MA(3) = 3.94$

Moving Average Smoothing

Year	Actual	MA(3)	Rate	Actual	MA(3)
1991	5.38	3.94	2000	5.82	
1992	3.43		2001	3.40	
1993	3.00		2002	1.61	
1994	4.25		2003	1.01	
1995	5.49		2004	1.37	
1996	5.01		2005	3.15	
1997	5.06		2006	4.73	
1998	4.78		2007	4.36	
1999	4.64		2008	1.37	

Moving Average Smoothing

Year	Actual	MA(3)	Rate	Actual	MA(3)
1991	5.38		2000	5.82	
1992	3.43	3.94	2001	3.40	
1993	3.00	3.56	2002	1.61	
1994	4.25		2003	1.01	
1995	5.49		2004	1.37	
1996	5.01		2005	3.15	
1997	5.06		2006	4.73	
1998	4.78		2007	4.36	
1999	4.64		2008	1.37	

Moving Average Smoothing

Year	Actual	MA(3)	Rate	Actual	MA(3)
1991	5.38		2000	5.82	4.62
1992	3.43	3.94	2001	3.40	3.61
1993	3.00	3.56	2002	1.61	2.01
1994	4.25	4.25	2003	1.01	1.33
1995	5.49	4.92	2004	1.37	1.84
1996	5.01	5.19	2005	3.15	3.08
1997	5.06	4.95	2006	4.73	4.08
1998	4.78	4.83	2007	4.36	3.49
1999	4.64	5.08	2008	1.37	

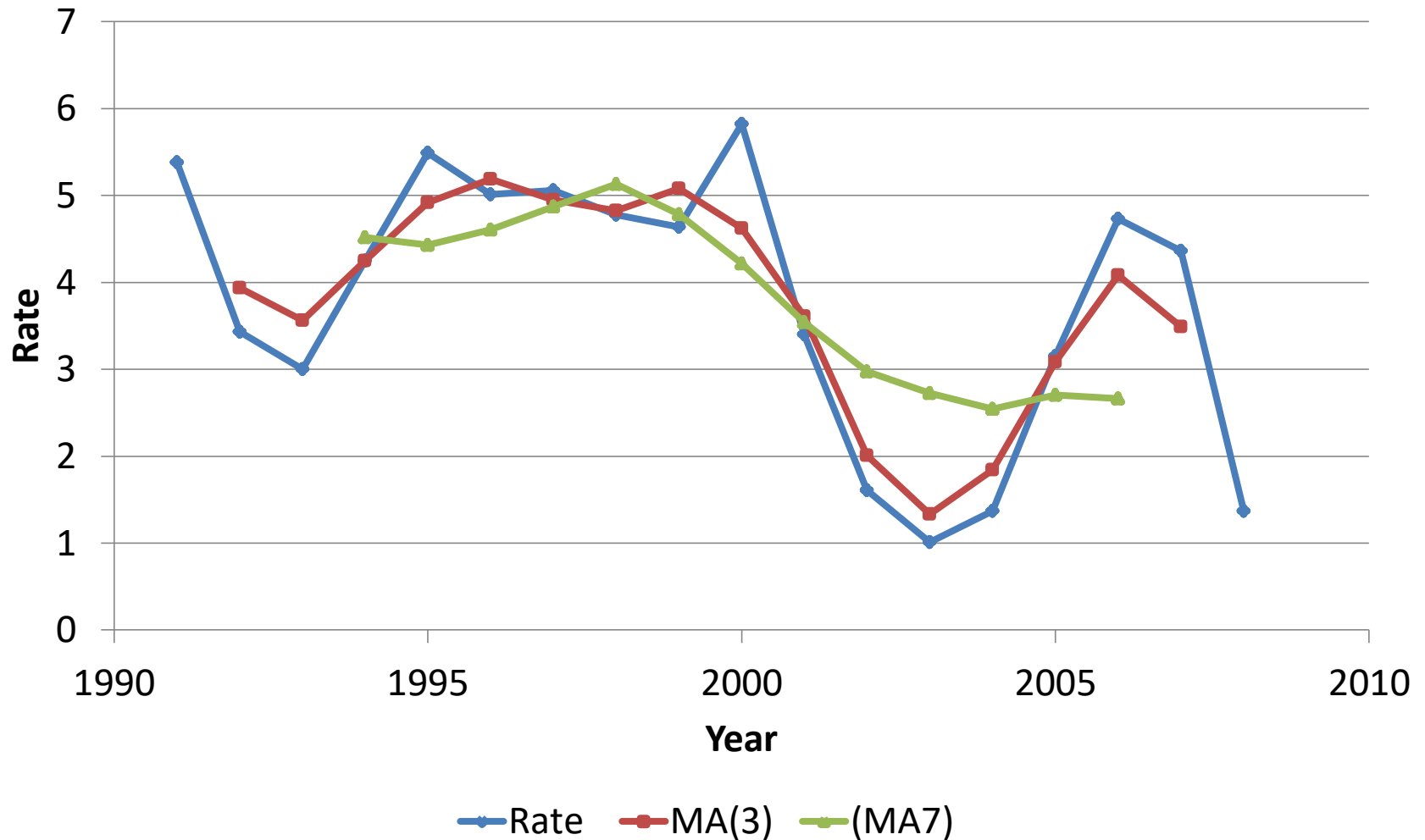
Exercise

- Coba tentukan rata-rata bergerak 7 tahunan untuk data yang telah diberikan!

Exercise (Answer)

Year	Actual	MA(7)	Rate	Actual	MA(7)
1991	5.38		2000	5.82	4.22
1992	3.43		2001	3.40	3.54
1993	3.00		2002	1.61	2.98
1994	4.25	4.52	2003	1.01	2.73
1995	5.49	4.43	2004	1.37	2.55
1996	5.01	4.60	2005	3.15	2.71
1997	5.06	4.87	2006	4.73	2.67
1998	4.78	5.13	2007	4.36	
1999	4.64	4.79	2008	1.37	

Plot of Three-Month U.S. Treasury Bill Rate, 1991 through 2008



Moving Average Smoothing

- If cyclical fluctuations are present in the data, choose L that corresponds to the **estimated length of a cycle** in the series.
- For annual time-series data that has no obvious cyclical fluctuations, most people choose **three** years, **five** years, or **seven** years.
- Selecting moving averages that are longer than seven years is usually undesirable, because too many moving average values are missing.

Linear Trend Model

- Trend linear model is used when a straight-line trend adequately fits the data.
- The method we used in developing the linear trend model here is least-square method (metode kuadrat terkecil)

Linear Trend Model

- Equation:

$$\hat{Y}_i = b_0 + b_1 X_i$$

where

\hat{Y}_i : predicted value of Y for observation
i

X_i : value of X for observation i

b_0 : sample Y intercept

b_1 : sample slope

Linear Trend Model

$$\hat{Y}_i = b_0 + b_1 X_i$$

$$b_1 = \frac{\sum_{i=1}^n X_i Y_i - \frac{\left(\sum_{i=1}^n X_i\right)\left(\sum_{i=1}^n Y_i\right)}{n}}{\sum_{i=1}^n X_i^2 - \frac{\left(\sum_{i=1}^n X_i\right)^2}{n}}$$
$$b_0 = \bar{Y} - b_1 \bar{X}$$

Example

Revenues (in Billions of Dollars) for the Coca-Cola Company

Year	Revenue	Year	Revenue
1995	18.0	2003	21.0
1996	18.5	2004	21.9
1997	18.9	2005	23.1
1998	18.8	2006	24.1
1999	19.8	2007	28.9
2000	20.5	2008	31.9
2001	20.1	2009	31.0
2002	19.6		

Example (Answer)

Year	Revenue (Y_i)	X_i	$X_i \cdot Y_i$	$(X_i)^2$
1995	18.0	0	0	0
1996	18.5	1	18.5	1
1997	18.9	2	37.8	4
1998	18.8	3	56.4	9
1999	19.8	4	79.2	16
2000	20.5	5	102.5	25
2001	20.1	6	120.6	36
2002	19.6	7	137.2	49
2003	21.0	8	168	64
2004	21.9	9	197.1	81
2005	23.1	10	231	100
2006	24.1	11	265.1	121
2007	28.9	12	346.8	144
2008	31.9	13	414.7	169
2009	31.0	14	434	196
Total	336.1	105	2608.9	1015

Example (Answer)

Year	Revenue (Y_i)	X_i	$X_i \cdot Y_i$	$(X_i)^2$
1995	18.0	0	0	0
1996	18.5	1	18.5	1
1997	18.9	2	37.8	4
1998	18.8	3	56.4	9
1999	19.8	4	79.2	16
2000	20.5	5	102.5	25
2001	20.1	6	120.6	36
2002	19.6	7	137.2	49
2003	21.0	8	168.0	64
2004	21.9	9	197.1	81
2005	23.1	10	231	100
2006	24.1	11	265.1	121
2007	28.9	12	346.8	144
2008	31.9	13	414.7	169
2009	31.0	14	434	196
Total	336.1	105	2608.9	1015

$\bar{Y} = 22.41$

$\bar{X} = 7$

Example (Answer)

$$b_1 = \frac{\sum_{i=1}^n X_i Y_i - \frac{\left(\sum_{i=1}^n X_i\right)\left(\sum_{i=1}^n Y_i\right)}{n}}{\sum_{i=1}^n X_i^2 - \frac{\left(\sum_{i=1}^n X_i\right)^2}{n}}$$

Example (Answer)

$$b_1 = \frac{2608.9 - \frac{(105)(336.1)}{15}}{1015 - \frac{(105)^2}{15}}$$

$$b_1 = \frac{2608.9 - 2352.7}{1015 - 735} = \frac{256.2}{280} = 0.915$$

Example (Answer)

$$b_0 = \bar{Y} - b_1 \bar{X}$$

$$b_0 = 22.41 - 0.915 \times 7$$

$$b_0 = 22.41 - 6.405 = 16.002$$

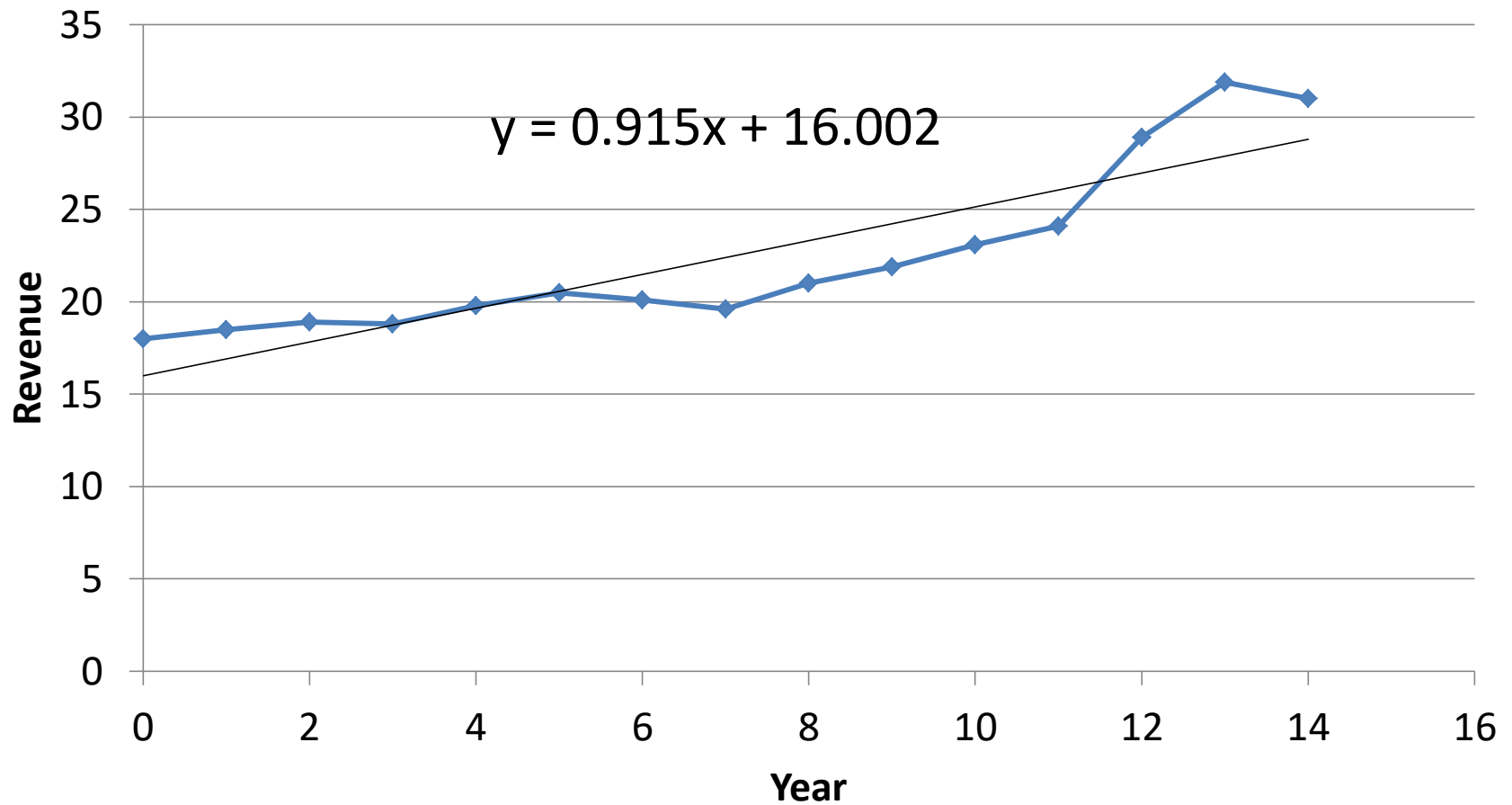
Example (Answer)

$$b_1 = 0.915 \qquad b_0 = 16.002$$

$$\hat{Y}_i = b_0 + b_1 X_i$$

$$\hat{Y}_i = 16.002 + 0.915 X_i$$

Plot of Revenues (in Billions of Dollars) for the Coca-Cola Company



EXERCISE

16.5

Data berikut ini merupakan jumlah kecelakaan yang terjadi pada kejuaraan NASCAR Sprint Cup dari tahun 2001 hingga tahun 2009

- a. Hitunglah rata-rata bergerak 3 tahunan dan 5 tahunan.
- b. Bagaimana tren jumlah kecelakaan pada kejuaraan NASCAR?

Tahun	Jumlah Kecelakaan
2001	200
2002	186
2003	235
2004	204
2005	253
2006	237
2007	240
2008	211
2009	195

16.16

Data berikut menunjukkan jumlah energi listrik (dalam megawatt) yang dihasilkan oleh pembangkit listrik tenaga matahari setiap tahun di U.S. dari tahun 2000 hingga tahun 2008

Tahun	Jumlah Energi Listrik
2000	18
2001	27
2002	44
2003	68
2004	83
2005	100
2006	140
2007	210
2008	250

16.16

- a. Hitunglah model peramalan trend linear dari data tersebut dengan menggunakan metode kuadrat terkecil!
- b. Tentukan peramalan jumlah energi listrik yang dihasilkan oleh pembangkit listrik tenaga matahari pada tahun 2009 dan 2010.

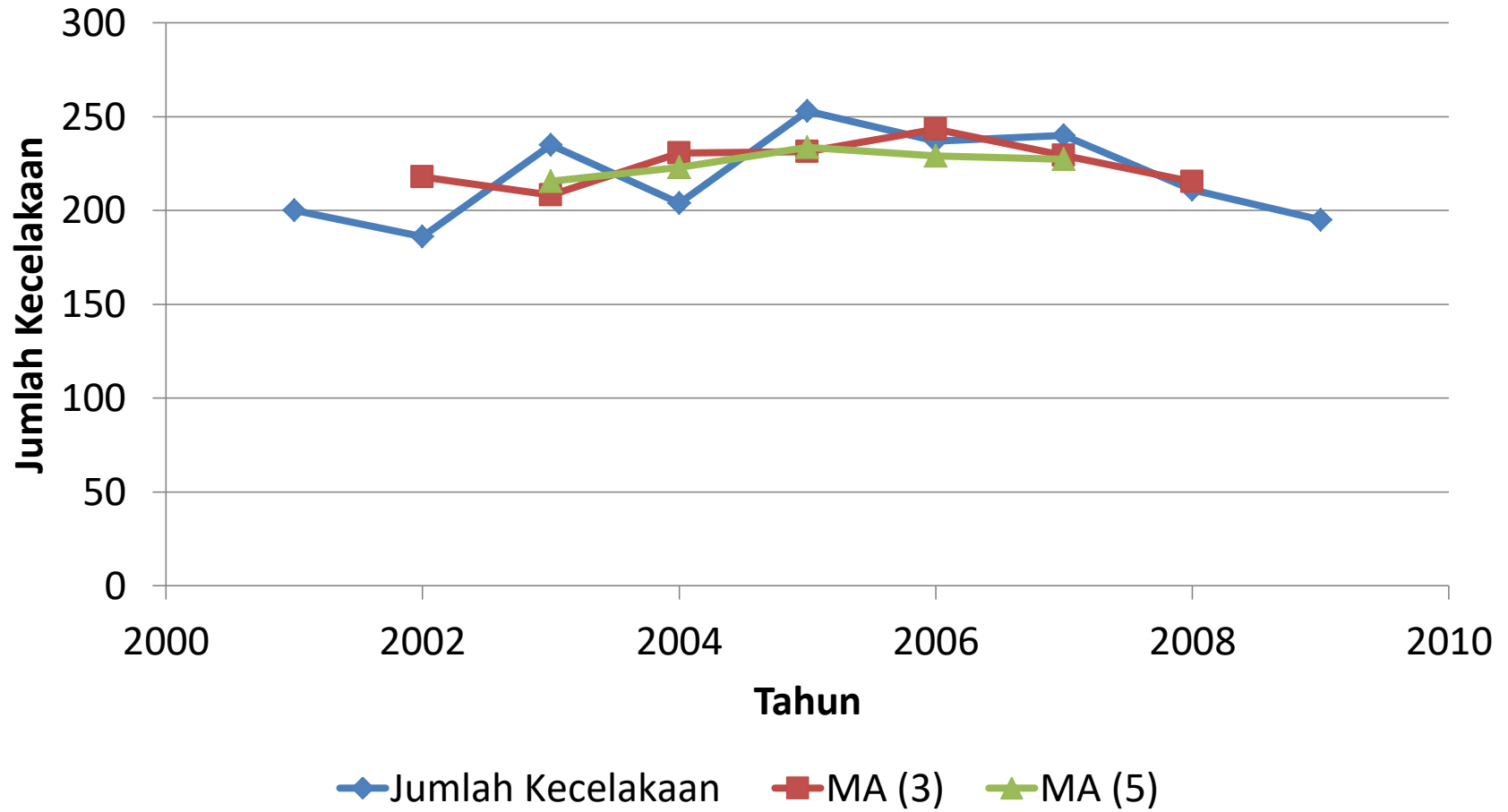
Tahun	Jumlah Energi Listrik
2000	18
2001	27
2002	44
2003	68
2004	83
2005	100
2006	140
2007	210
2008	250

ANSWER

16.5

Tahun	Jumlah Kecelakaan	MA (3)	MA (5)
2001	200		
2002	186	207	
2003	235	208	215.6
2004	204	231	223
2005	253	231	233.8
2006	237	243	229
2007	240	229	227.2
2008	211	215	
2009	195		

16.5



16.16

Tahun	Jumlah Energi Listrik (Y _i)	X _i	X _i Y _i	(X _i) ²
2000	18	0	0	0
2001	27	1	27	1
2002	44	2	88	4
2003	68	3	204	9
2004	83	4	332	16
2005	100	5	500	25
2006	140	6	840	36
2007	210	7	1470	49
2008	250	8	2000	64
Total	940	36	5461	204
Average	104.44	4.00		

16.16

$$b_1 = \frac{5461 - \frac{36.940}{9}}{204 - \frac{36^2}{9}}$$

$$b_1 = \frac{5461 - 3760}{204 - 144} = \frac{1701}{60} = 28.35$$

16.16

$$b_0 = \bar{Y} - b_1 \bar{X}$$

$$b_0 = 104.44 - 28.35 \times 4$$

$$b_0 = 104.44 - 113.4 = -8.9556$$

16.16

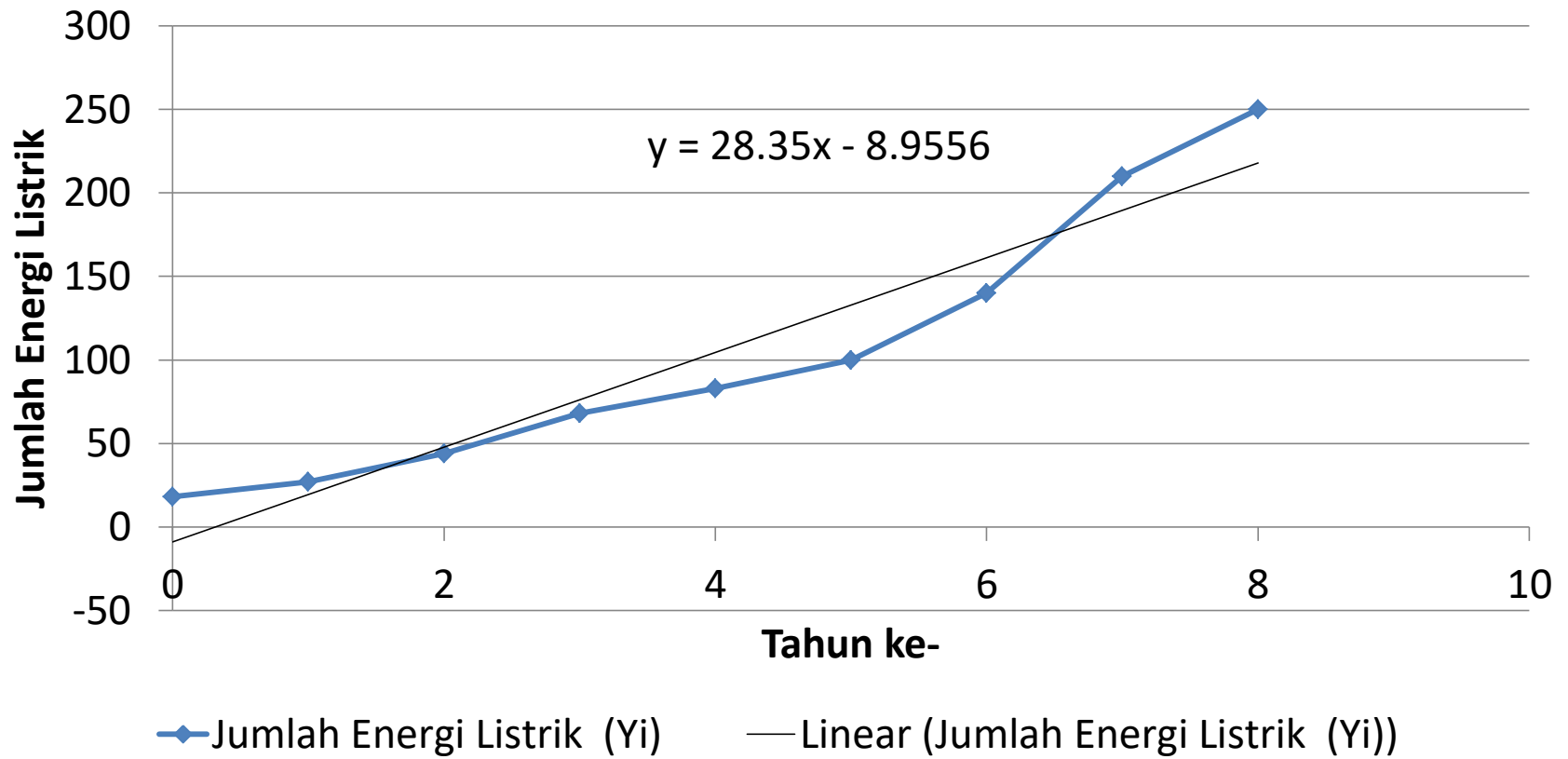
$$b_1 = 28.35 \qquad b_0 = -8.9556$$

$$\hat{Y}_i = b_0 + b_1 X_i$$

$$\hat{Y}_i = 28.35 X_i - 8.9556$$

16.16

Jumlah Energi Listrik (Yi)



THANK YOU