Business Statistic

Week 4

Basic Probability

Agenda

Time	Activity
50 minutes	Basic Probability
50 minutes	Conditional Probability and Bayes Theorem

Objectives

By the end of this class student should be able to:

- Understand different types of probabilities
- Compute probabilities
- Revise probabilities in light of new information

Basic Probability Concepts

- Probability – the chance that an uncertain event will occur (always between 0 and 1)

- Impossible Event – an event that has no chance of occurring (probability = 0)

- Certain Event – an event that is sure to occur (probability = 1)

Assessing Probability

- There are three approaches to assessing the probability of an uncertain event:

1. a priori -- based on prior knowledge of the process

$$\text{probability of occurrence} = \frac{X}{T} = \frac{\text{number of ways the event can occur}}{\text{total number of elementary outcomes}}$$

Assuming all outcomes are equally likely

2. empirical probability

$$\text{probability of occurrence} = \frac{\text{number of ways the event can occur}}{\text{total number of elementary outcomes}}$$

3. subjective probability

based on a combination of an individual's past experience, personal opinion, and analysis of a particular situation

ANY COMMENT?

Events

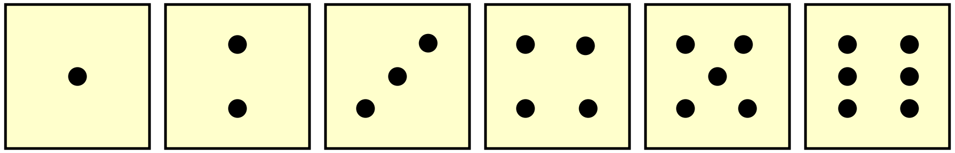
Each possible outcome of a variable is an event.

- Simple event
  - An event described by a single characteristic
  - e.g., A red card from a deck of cards
- Joint event
  - An event described by two or more characteristics
  - e.g., An ace that is also red from a deck of cards
- Complement of an event A (denoted A')
  - All events that are not part of event A
  - e.g., All cards that are not diamonds

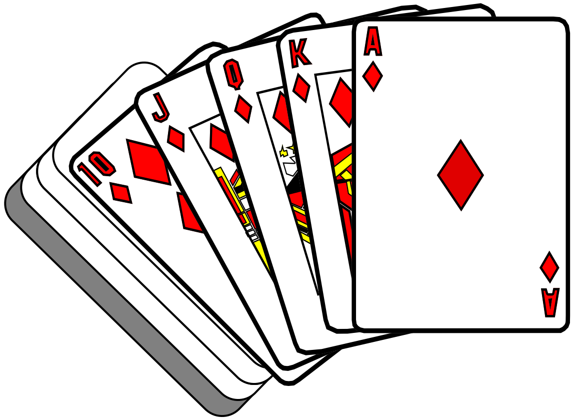
Sample Space

The Sample Space is the collection of all possible events

- e.g. All 6 faces of a dice:



- e.g. All 52 cards of a bridge deck:



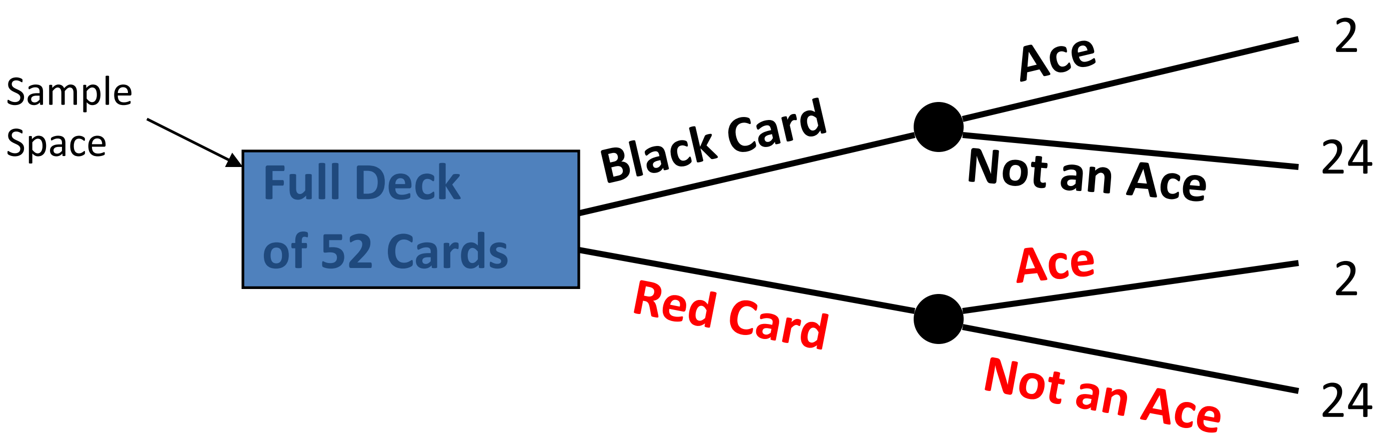
Visualizing Events

- Contingency Tables

	Ace	Not Ace	Total
Black	2	24	26
Red	2	24	26
Total	4	48	52

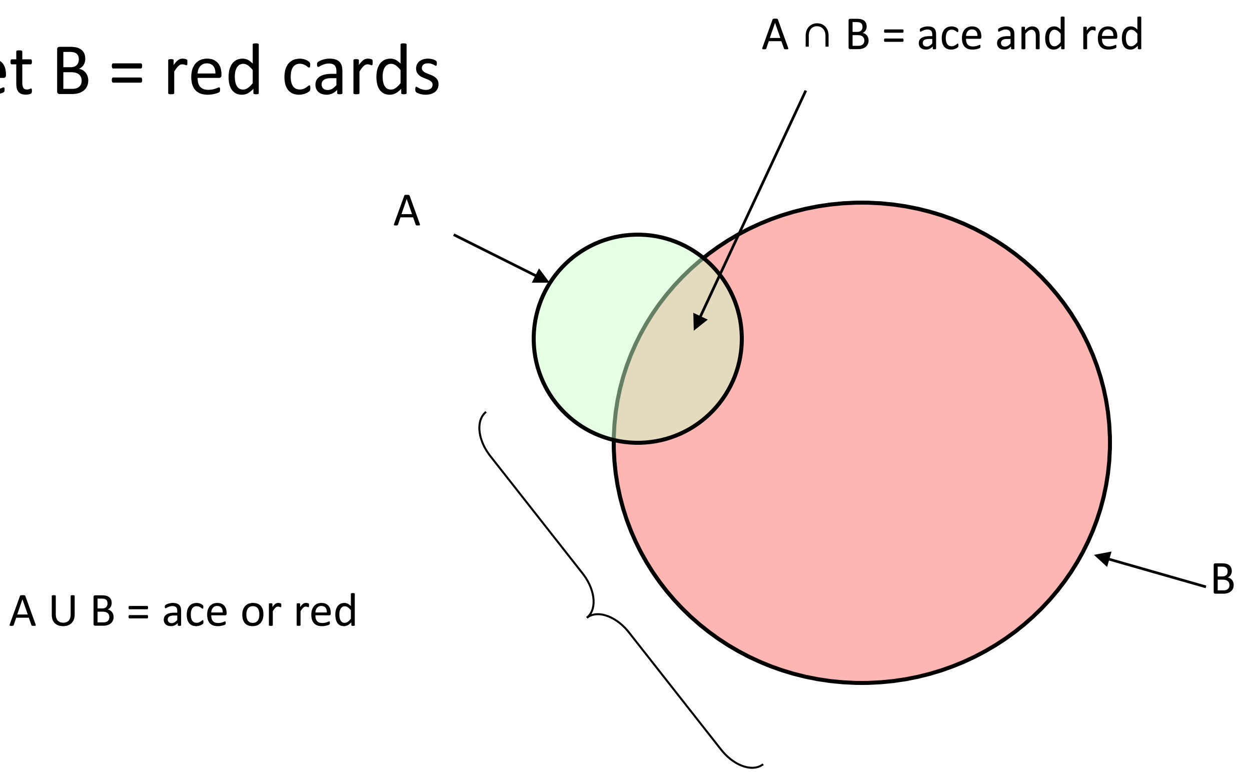
Sample Space

- Decision Trees



Visualizing Events

- Venn Diagrams
  - Let A = aces
  - Let B = red cards



ANY QUESTION?

Mutually Exclusive Events

- Mutually exclusive events
  - Events that cannot occur simultaneously

Example: Drawing one card from a deck of cards

A = queen of diamonds; B = queen of clubs

  - Events A and B are mutually exclusive

Collectively Exhaustive Events

- Collectively exhaustive events
  - One of the events must occur
  - The set of events covers the entire sample space

example:

A = aces; B = black cards;
C = diamonds; D = hearts

  - Events A, B, C and D are collectively exhaustive (but not mutually exclusive – an ace may also be a heart)
  - Events B, C and D are collectively exhaustive and also mutually exclusive

Exercise 1

For each of the following, state whether the events created are mutually exclusive and collectively exhaustive.

a. Registered voters in the United States were asked whether they are registered as Republicans or Democrats
b. Each respondent was classified by the type of car he or she drives: sedan, SUV, American, European, Asian, or none.
c. A product was classified as defective or not defective.

Exercise 1 (Answer)

a. Mutually exclusive, not collectively exhaustive.
b. Not mutually exclusive, not collectively exhaustive.
c. Mutually exclusive, collectively exhaustive.

PROBABILITIES…

Probability

Gender	Has sibling(s)?		Total
	Yes	No	
Male	6	1	7
Female	18	2	20
Total	24	3	27

Gender	Has sibling(s)?		Total
	Yes	No	
Male	0.22	0.04	0.26
Female	0.67	0.07	0.74
Total	0.89	0.11	1

Computing Joint and Marginal Probabilities

- The probability of a joint event, A and B:

$$P(\text{A and B}) = \frac{\text{number of outcomes satisfying A and B}}{\text{total number of elementary outcomes}}$$

- Computing a marginal (or simple) probability:

$$P(A) = P(\text{A and } B_1) + P(\text{A and } B_2) + \Lambda + P(\text{A and } B_k)$$

  - Where B1, B2, …, Bk are k mutually exclusive and collectively exhaustive events

Marginal & Joint Probabilities In A Contingency Table

Event	Event		Total
	B_1	B_2	
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	$P(A_1)$
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	**1**

Joint Probabilities

Marginal (Simple) Probabilities

General Addition Rule

General Addition Rule:

P(A or B) = P(A) + P(B) - P(A and B)

If A and B are mutually exclusive, then

P(A and B) = 0, so the rule can be simplified:

P(A or B) = P(A) + P(B)

For mutually exclusive events A and B

Exercise 2

A sample of 500 respondents in a large metropolitan area was selected to study consumer behavior. Among the questions asked was “Do you enjoy shopping for clothing?” Of 240 males, 136 answered yes. Of 260 females, 224 answered yes. Construct a contingency table to evaluate the probabilities.

Exercise 2

What is the probability that a respondent chosen at random

a. enjoys shopping for clothing?
b. is a female *and* enjoys shopping for clothing?
c. is a female *or* enjoys shopping for clothing?
d. is a male *or* a female?

Exercise 2 (Answer)

	Enjoy shopping for clothing		
Gender	Yes	No	Total
Male	136	104	240
Female	224	36	260
Total	360	140	500

	Enjoy shopping for clothing		
Male	Yes	No	Total
Female	0.27	0.21	0.48
	0.45	0.07	0.52
Total	0.72	0.28	1.00

Exercise 2 (Answer)

a. enjoys shopping for clothing = 0.72
b. is a female *and* enjoys shopping for clothing = 0.45
c. is a female *or* enjoys shopping for clothing = 0.79
d. is a male *or* a female = 1

CONDITIONAL PROBABILITIES

Computing Conditional Probabilities

- A conditional probability is the probability of one event, given that another event has occurred:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

→ The conditional probability of A given that B has occurred

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

→ The conditional probability of B given that A has occurred

Where P(A and B) = joint probability of A and B

P(A) = marginal or simple probability of A

P(B) = marginal or simple probability of B

Independence

- Two events are independent if and only if:

$$P(A \mid B) = P(A)$$

- Events A and B are independent when the probability of one event is not affected by the fact that the other event has occurred

Multiplication Rules

- Multiplication rule for two events A and B:

$$P(A \text{ and } B) = P(A|B)P(B)$$

Note: If A and B are independent, then

$$P(A|B) = P(A)$$

and the multiplication rule simplifies to

$$P(A \text{ and } B) = P(A)P(B)$$

Bayes' Theorem

- Bayes' Theorem is used to revise previously calculated probabilities based on new information.

- Developed by Thomas Bayes in the 18^{th} Century.

- It is an extension of conditional probability.

Bayes' Theorem

$$P(B_i | A) = \frac{P(A | B_i)P(B_i)}{P(A | B_1)P(B_1) + P(A | B_2)P(B_2) + \cdots + P(A | B_k)P(B_k)}$$

- where:

  B_i = i^{th} event of k mutually exclusive and collectively exhaustive events

  A = new event that might impact $P(B_i)$

Exercise 3

From the problem in Exercise 2.

a. Suppose that the respondent chosen is a female. What is the probability that she does not enjoy shopping for clothing?
b. Suppose that the respondent chosen enjoys shopping for clothing. What is the probability that the individual is a male?
c. Are enjoying shopping for clothing and the gender of the individual independent? Explain.

Exercise 3 (Answer)

	Enjoy shopping for clothing		
Gender	Yes	No	Total
Male	0.27	0.21	0.48
Female	0.45	0.07	0.52
Total	0.72	0.28	1.00

Let’s take :

P(E) = P(Enjoy shopping for clothing)

Thus **P(B’)** = P(Not enjoy shopping for clothing)

P(G) = P(Male)

Thus **P(K’)** = P(Female)

Exercise 3 (Answer)

a. $P(E'|G') = 0.07 / 0.52 = 0.14$

b. $P(G|E) = 0.27 / 0.72 = 0.38$

c. Two events are considered independent if:

$$P(A|B) = P(A)$$

In this case:

$P(G|E) = P(G)$

$0.38 \neq 0.48$

→ Therefore, gender and enjoy shopping for clothing are not independent

Exercise 4

Olive Construction Company is determining whether it should submit a bid for a new shopping center. In the past, Olive's main competitor, Base Construction Company, has submitted bids 70% of the time. If Base Construction Company does not bid on a job, the probability that Olive Construction Company will get the job is 0.50. If Base Construction Company bids on a job, the probability that Olive Construction Company will

get the job is 0.25.

Exercise 4

a. If Olive Construction Company gets the job, what is the probability that Base Construction Company did not bid?
b. What is the probability that Olive Construction Company will get the job?

Exercise 4 (Answer)

	Olive		
Base	Win	Lose	Total
Submit			0.70
Not submit			0.30
Total			1.00

- P(Olive|Base’) = 0.5
  - P(Olive & Base’) = P(Olive|Base’) x P(Base’) = 0.15

Exercise 4 (Answer)

	Olive		
Base	Win	Lose	Total
Submit			0.70
Not submit	0.15	0.15	0.30
Total			1.00

- P(Olive|Base) = 0.25
  - P(Olive & Base) = P(Olive|Base) x P(Base) = 0.175

Exercise 4 (Answer)

	Olive		
Base	Win	Lose	Total
Submit	0.175	0.525	0.70
Not submit	0.15	0.15	0.30
Total	0.325	0.675	1.00

a. P(Base’|Olive) = 0.15 / 0.325 = 0.46
b. P(Olive) = 0.325

DISCRETE VS. CONTINUOUS

Probability

- Discrete Probability

- Continuous Probability

EXERCISE

4.60

A survey by the Pew Research Center indicated that 81% of 18- to 25-year-olds had getting rich as a goal, as compared to 62% of 26- to 40-year-olds. Suppose that the survey was based on 500 respondents from each of the two groups.

a. Construct a contingency table.
b. Give an example of a simple event and a joint event.
c. What is the probability that a randomly selected respondent has a goal of getting rich?
d. What is the probability that a randomly selected respondent has a goal of getting rich *and* is in the 26- to 40-year-old group?
e. Are the events “age group” and “has getting rich as a goal” independent? Explain.

4.61

The owner of a restaurant serving Continental-style entrees was interested in studying ordering patterns of patrons for the Friday-to-Sunday weekend time period. Records were maintained that indicated the demand for dessert during the same time period. The owner decided to study two other variables, along with whether a dessert was ordered: the gender of the individual and whether a beef entree was ordered. The results are as follows:

4.61

	GENDER		
DESSERT ORDERED	**Male**	**Female**	**Total**
Yes	96	40	136
No	224	240	464
Total	**320**	**280**	**600**

	BEEF ENTRÉE		
DESSERT ORDERED	**Yes**	**No**	**Total**
Yes	71	65	136
No	116	348	464
Total	**187**	**413**	**600**

4.61

A waiter approaches a table to take an order for dessert. What is the probability that the first customer to order at the table

a. orders a dessert?
b. orders a dessert *or* has ordered a beef entree?
c. is a female *and* does not order a dessert?
d. is a female *or* does not order a dessert?
e. Suppose the first person from whom the waiter takes the dessert order is a female. What is the probability that she does not order dessert?
f. Are gender and ordering dessert independent?
g. Is ordering a beef entree independent of whether the person orders dessert?

4.62

Which meal are people most likely to order at a drive through? A survey was conducted in 2009, but the sample sizes were not reported. Suppose the results, based on a sample of 100 males and 100 females, were as follows:

MEAL	Male	Female	Total
Breakfast	18	10	28
Lunch	47	52	99
Dinner	29	29	58
Snack/beverage	6	9	15
Total	**100**	**100**	**200**

4.62

If a respondent is selected at random, what is the probability that he or she

a. prefers ordering lunch at the drive-through?
b. prefers ordering breakfast or lunch at the drive-through?
c. is a male *or* prefers ordering dinner at the drive-through?
d. is a male *and* prefers ordering dinner at the drive-through?
e. Given that the person selected is a female, what is the probability that she prefers ordering breakfast at the drive-through?

4.63

According to a Gallup Poll, companies with employees who are engaged with their workplace have greater innovation, productivity, and profitability, as well as less employee turnover. A survey of 1,895 workers in Germany found that 13% of the workers were engaged, 67% were not engaged, and 20% were actively disengaged. The survey also noted that 48% of engaged workers strongly agreed with the statement “My current job brings out my most creative ideas.” Only 20% of the not engaged workers and 3% of the actively disengaged workers agreed with this statement. If a worker is known to strongly agree with the statement “My current job brings out my most creative ideas,” what is the probability that the worker is engaged?

THANK YOU!!