Decision Making Theory

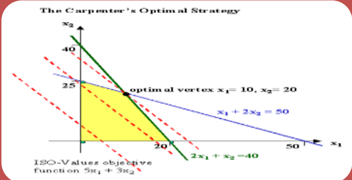
Week 2

Linear Programming

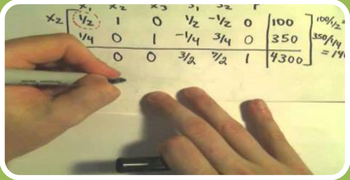
Graphic Method

Outlines

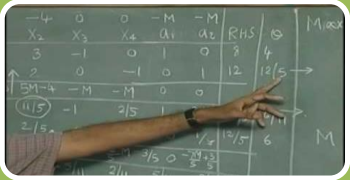
Linear Programing Model – Problem Formulation



Linear Programing Model – Graphic Solution



Linear Programing Model – Simplex Method (Maximize)



Linear Programing Model – Simplex Method (Minimize and Non-Standard)

Properties of Linear Programs

- One objective function
- One or more constraints
- Alternative course of action
  - Objective function and constraints are linear—proportionality and divisibility
  - Certainty
  - Divisibility
  - Nonnegative variables

The Steps in formulating a linear program

- Completely understand the managerial problem being faced
- Identify the objective and the constraints
- Define the decision variables
- Use the decision variables to write mathematical expressions for the objective function and the constraints

Developing Linear Programming Model

- Decision Variables
- Objective
- Objective Function
- Constraints
  - Nonnegative constraints

EXAMPLE

Flair Furniture Company

The Flair Furniture Company produces inexpensive tables and chairs. The production process for each is similar in that both require a certain number of hours of carpentry work and a certain number of labor hours in the painting and varnishing department. Each table takes 4 hours of carpentry and 2 hours in the painting and varnishing shop. Each chair requires 3 hours in carpentry and 1 hour in painting and varnishing.

Flair Furniture Company

During the current production period, 240 hours of carpentry time are available and 100 hours in painting and varnishing time are available. Each table sold yields a profit of $70; each chair produced is sold for a $50 profit.

Flair Furniture's problem is to determine the best possible combination of tables and chairs to manufacture in order to reach the maximum profit. The firm would like this production mix situation formulated as an LP problem.

ANSWER

PROBLEM FORMULATION

Problem formulation

- Objective: Maximize profit
- Constraints:
    1. The hours of carpentry time used cannot exceed 240 hours per week.
    2. The hours of painting and varnishing time used cannot exceed 100 hours per week.
- Decision variables:
    – T = Number of tables to be produced per week
    – C = Number of chairs to be produced per week

Problem formulation

- **Objective function:**

  Maximize profit = \$70T + \$50C

- **Carpentry department:**

  (4 hours per table)(Number of tables produced)
  + (3 hours per chair)(Number of chairs produced)

  Carpentry time used ≤ Carpentry time available

  $4T + 3C \leq 240$ (hours of carpentry time)

Problem formulation

- **Painting and varnishing department**

  (2 hours per table)(Number of tables produced)
  + (1 hours per chair)(Number of chairs produced)

  Painting and varnishing time used ≤ Painting and varnishing time available

  $2T + 1C \leq 100$ (hours of painting and varnishing time)

Problem formulation

- **Nonnegative constraints**

  To obtain meaningful solutions, the values for *T* and *C* must be nonnegative numbers

  $T \geq 0$ (number of tables produced is greater than or equal to 0)

  $C \geq 0$ (number of chairs produced is greater than or equal to 0)

Problem formulation

- Objective function:

  Maximize profit = $70T + $50C

- Constraints:

$$4T + 3C \leq 240$$

$$2T + 1C \leq 100$$

$$T \geq 0$$

$$C \geq 0$$

GRAPHIC SOLUTION

Graphical Representation of Constraints

$4T + 3C \leq 240$

- When T = 0:

  $4(0) + 3C = 240$

  $3C = 240$

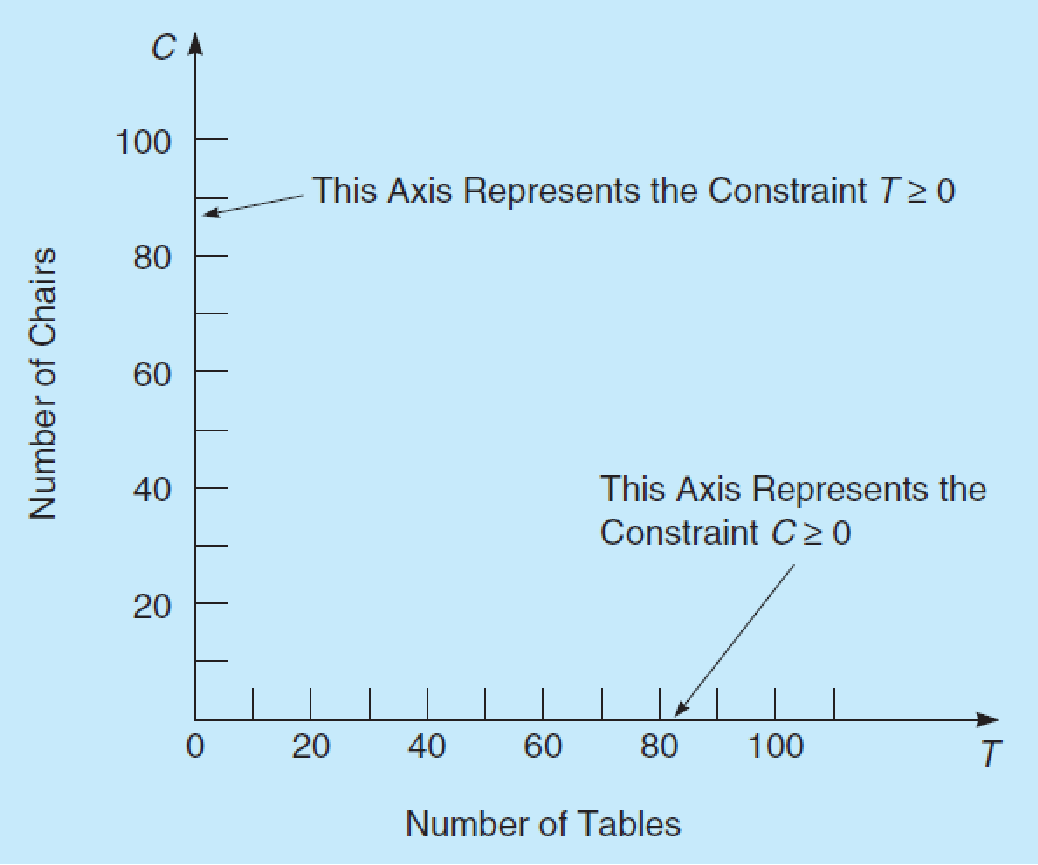
  $C = 80$

- When C = 0

  $4T + 3(0) = 240$

  $4T = 240$

  $T = 60$



Graphical Representation of Constraints

4T + 3C ≤ 240

- When T = 0:

  4(0) + 3C = 240

  3C = 240

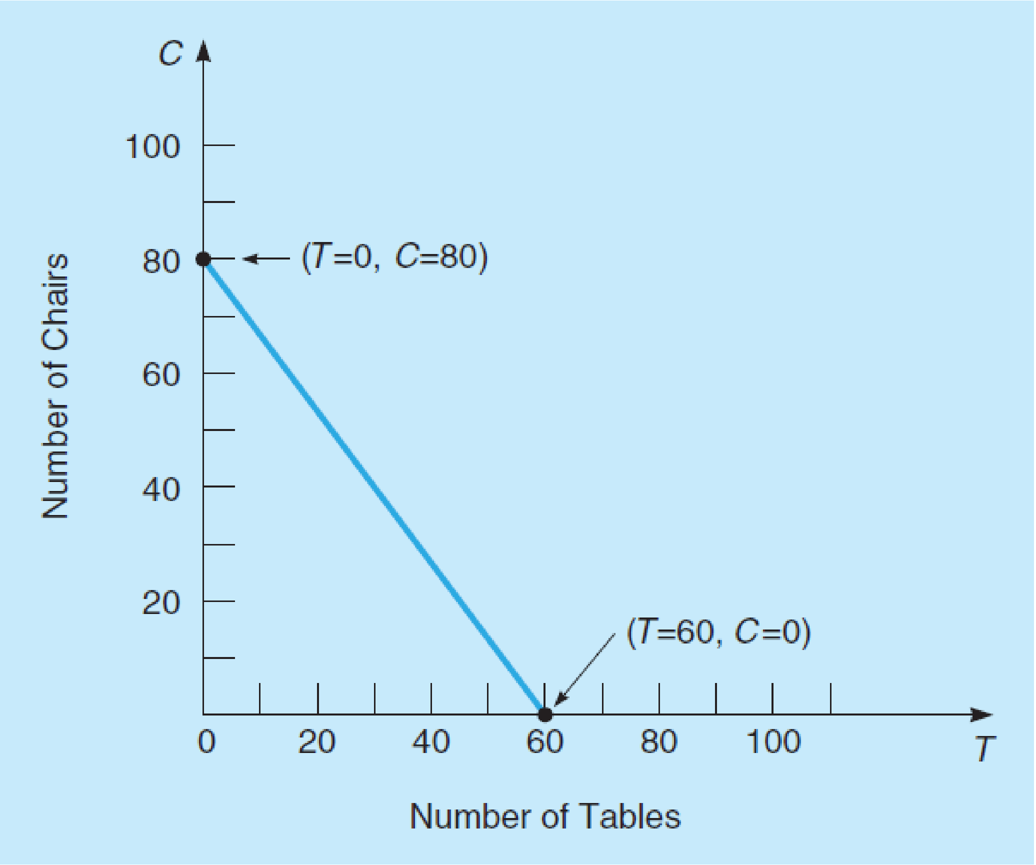
  C = 80

- When C = 0

  4T + 3(0) = 240

  4T = 240

  T = 60



Graphical Representation of Constraints

4T + 3C ≤ 240

- (30, 20)

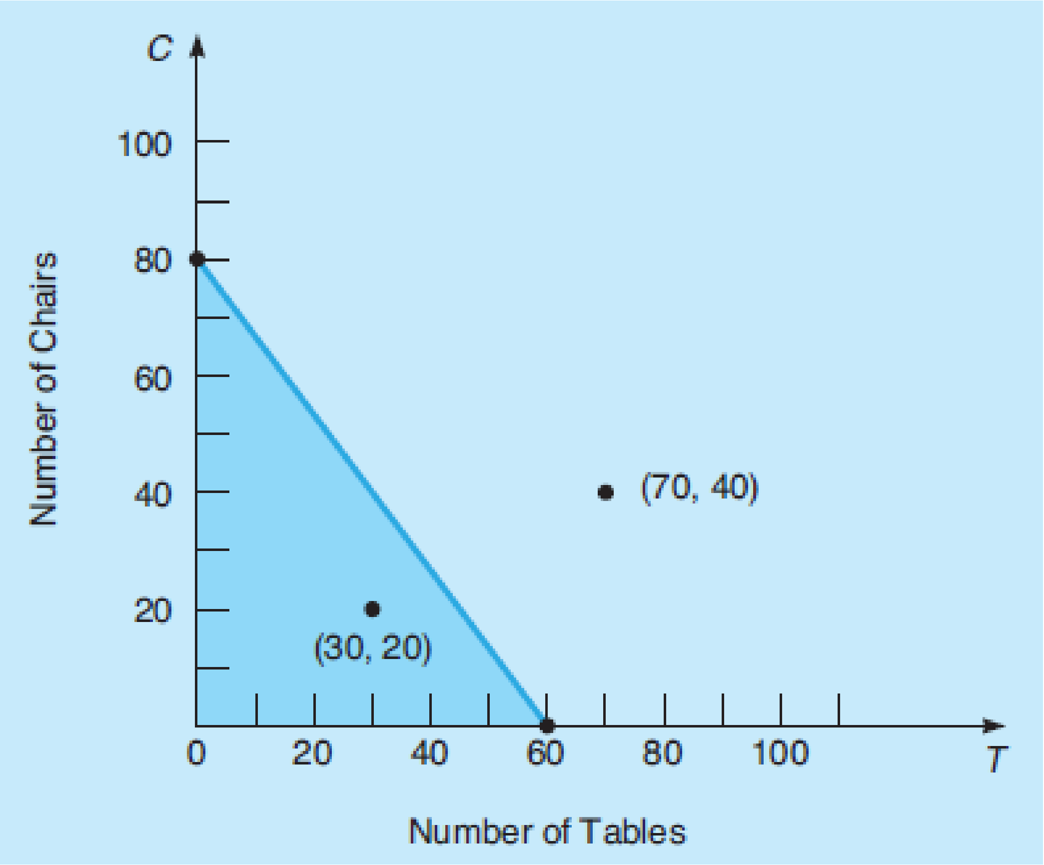
  4(30) + 3(20) = 180

  180 < 240

- (70, 40)

  4(70) + 3(40) = 400

  400 > 240



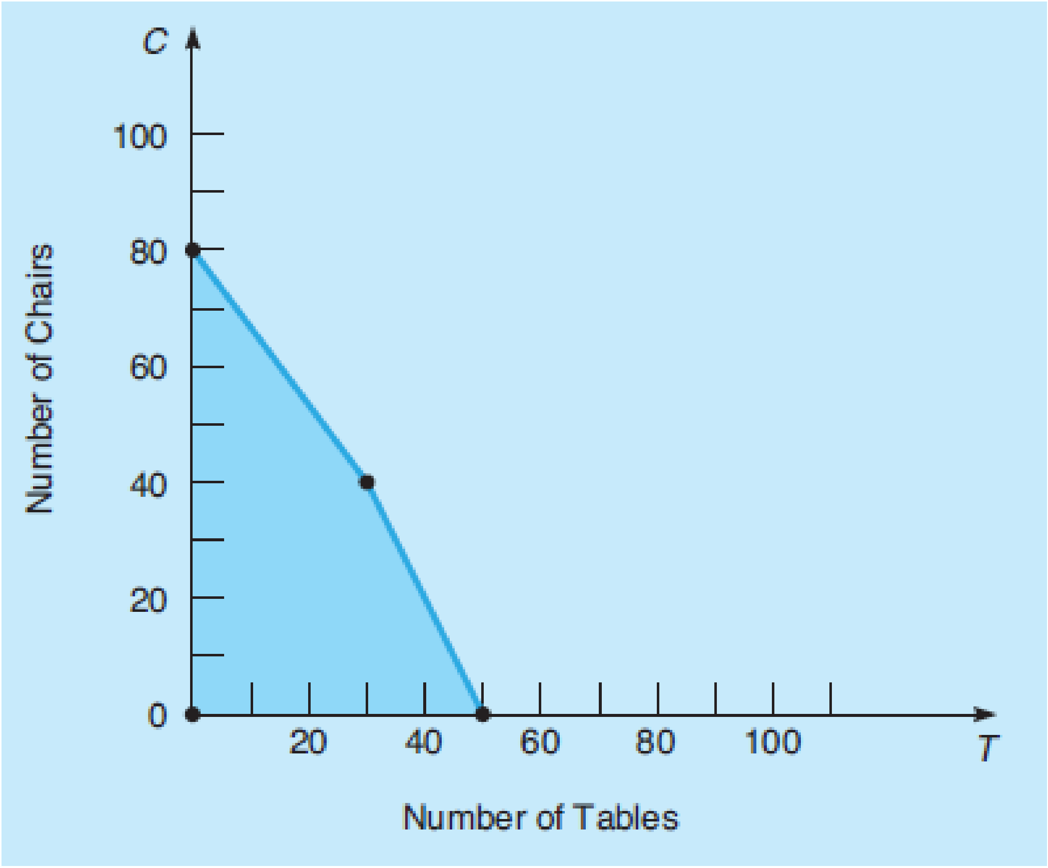
Corner Point Solution Method

$$4T + (3)(40) = 240$$

$$4T + 120 = 240$$

$$4T = 120$$

$$T = 30$$



Graphical Representation of Constraints

$2T + 1C \leq 100$

- When T = 0:

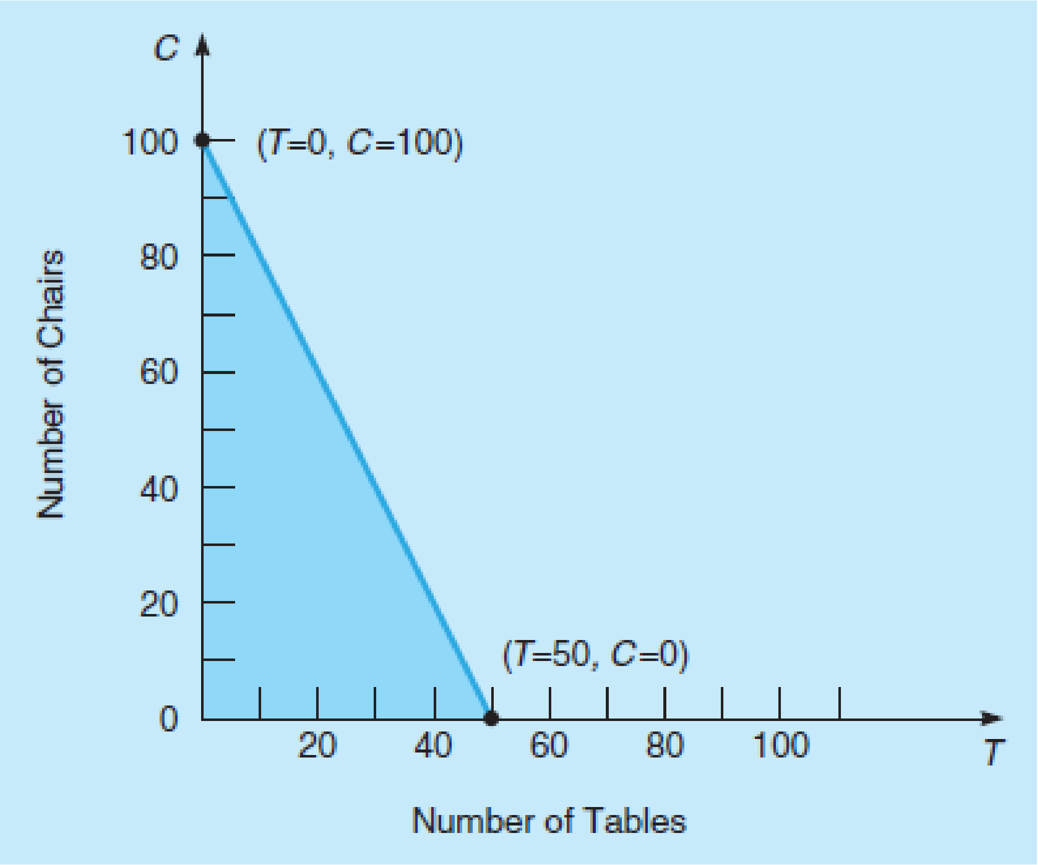
  $2(0) + 1C = 100$

  $C = 100$

- When C = 0

  $2T + 1(0) = 100$

  $T = 50$



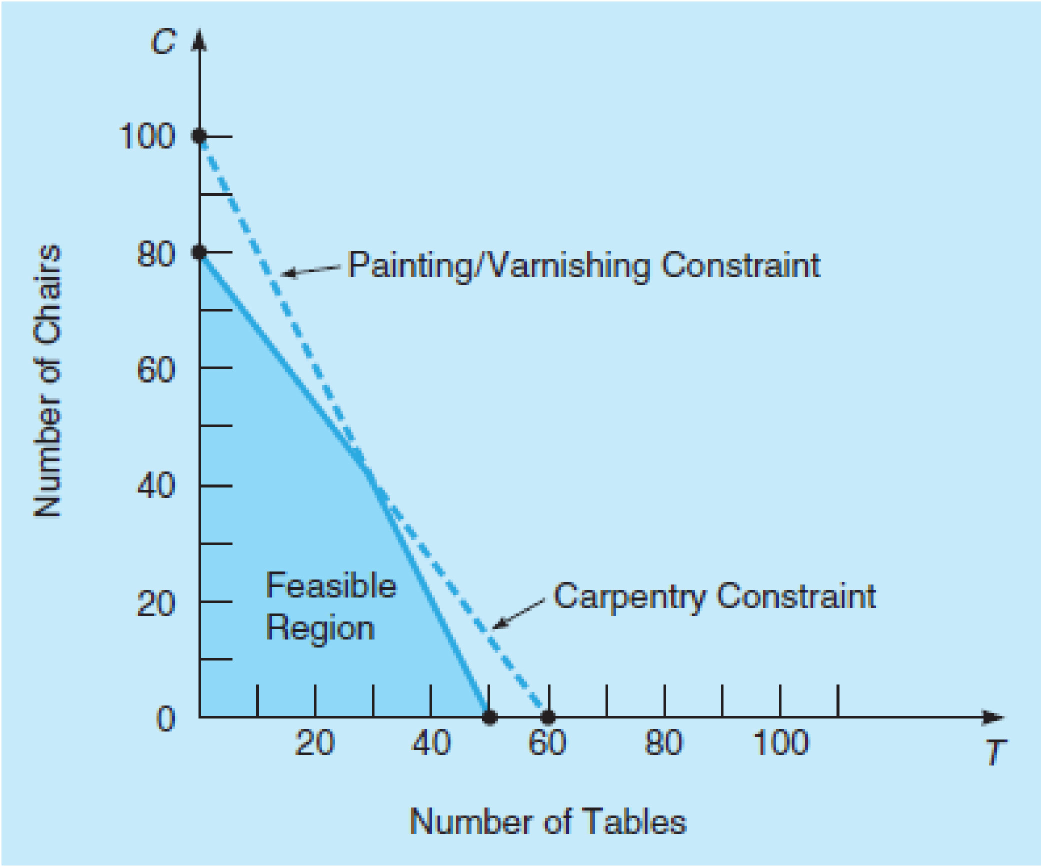
Graphical Representation of Constraints

$4T + 3C \leq 240$

$2T + 1\ C \leq 100$

$T \geq 0$

$C \geq 0$



Corner Point Solution Method

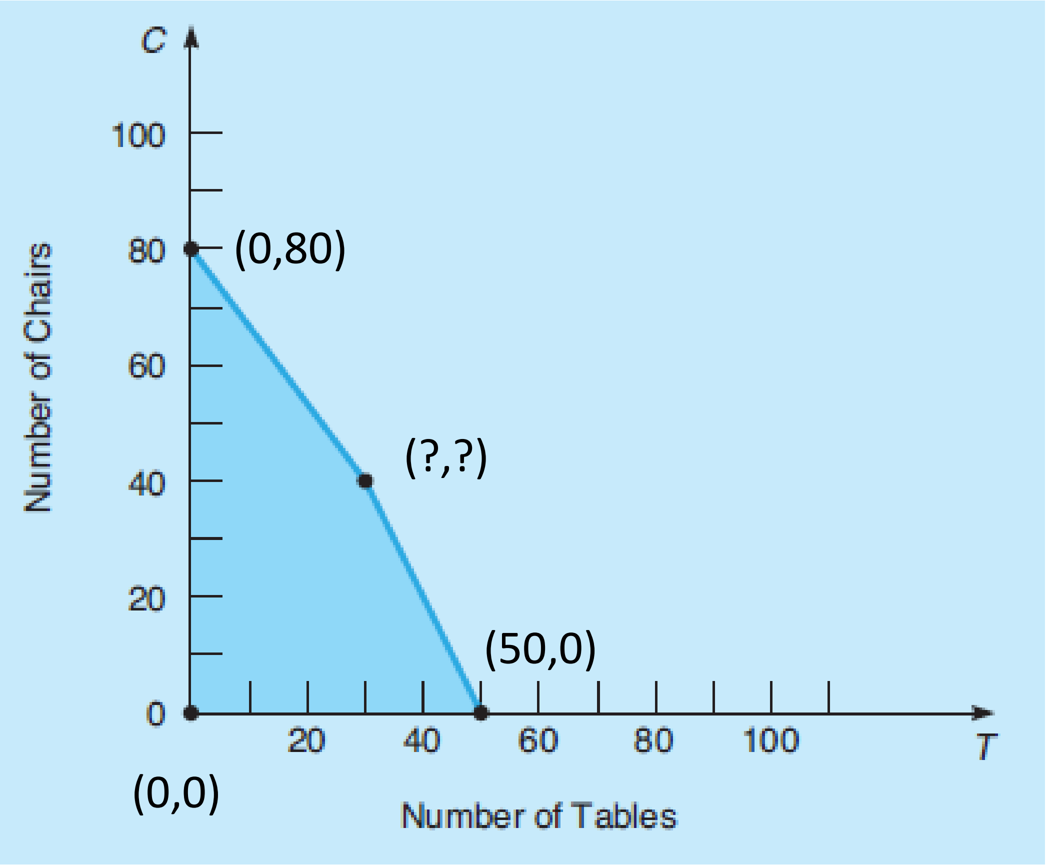
4T + 3C = 240

2T + 1C = 100 (-2)

4T + 3C = 240

<u>-4T - 2C = -200</u>

C = 40



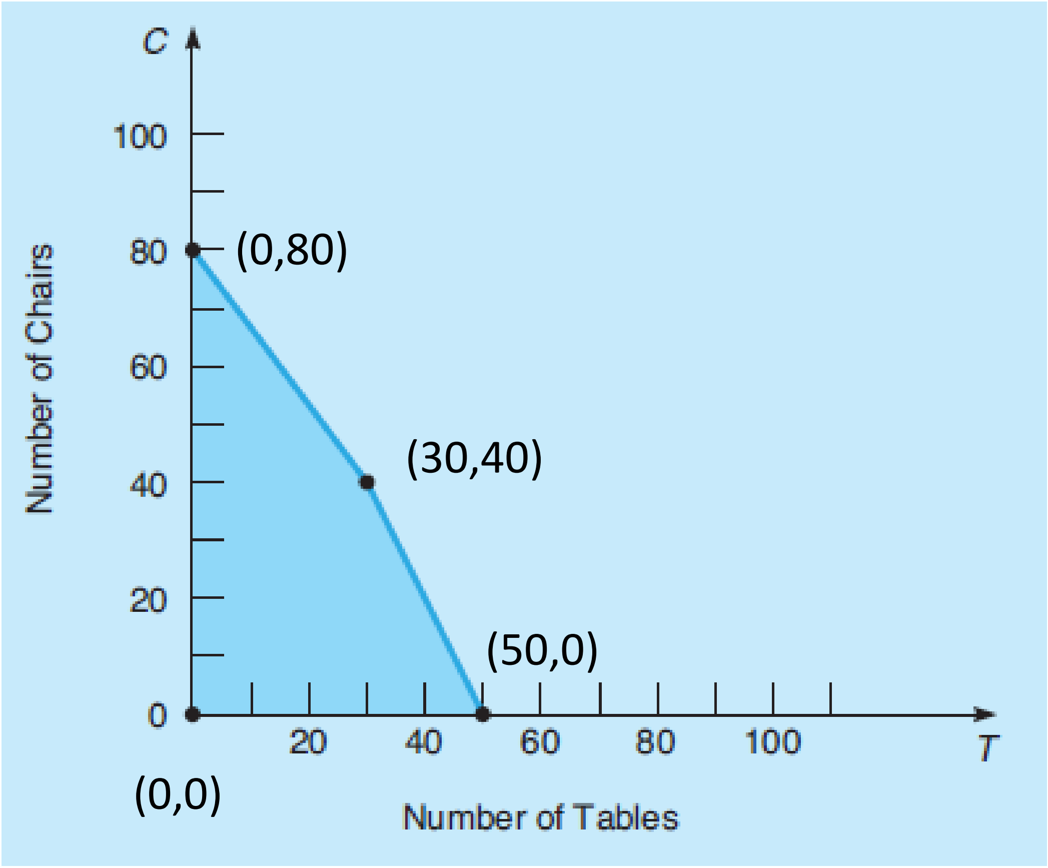
Corner Point Solution Method

$$4T + (3)(40) = 240$$

$$4T + 120 = 240$$

$$4T = 120$$

$$\mathbf{T = 30}$$



Corner Point Solution Method

Number of Tables (T)	Number of Chairs (C)	Profit = \$70T + \$50C
0	0	\$0
50	0	\$3,500
0	80	\$4,000
30	40	\$4,100

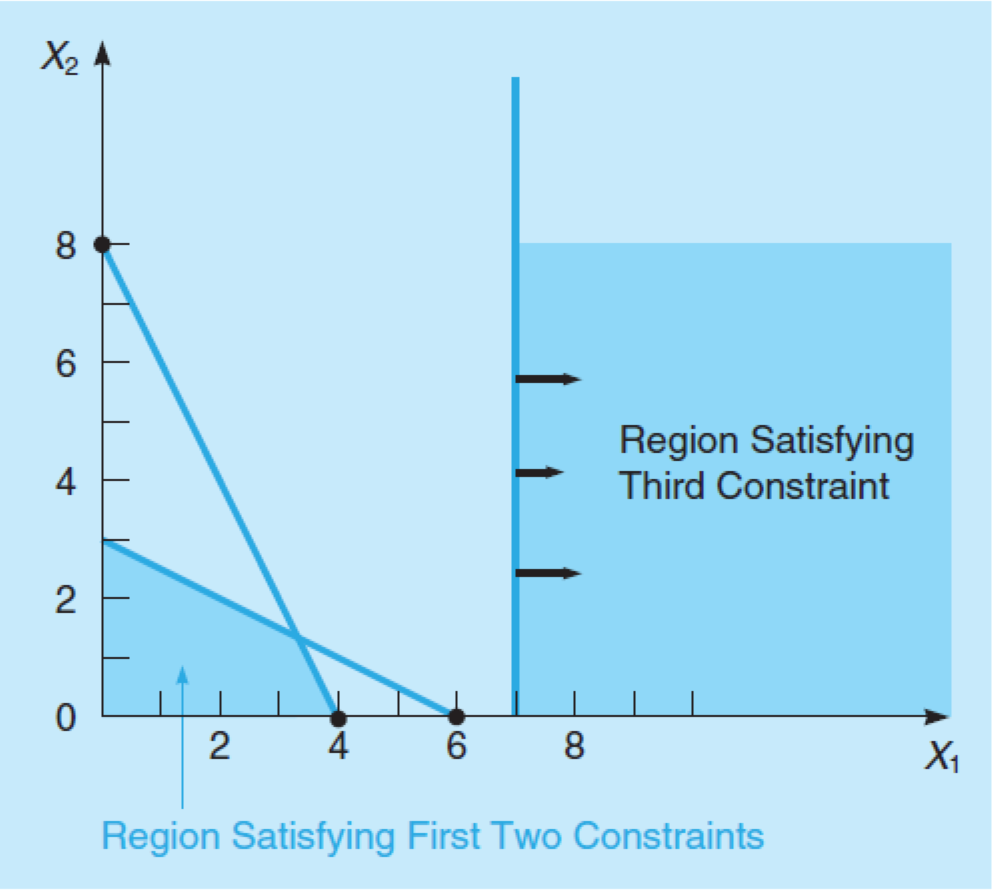
The highest **profit** is found to be **\$4,100**, which is obtained when **30 tables** and **40 chairs** are produced

Four Special Cases in LP

- No Feasible Solution
- Unboundedness
- Redundancy
- Alternate Optimal Solutions

No Feasible Solution

$$X_1 + 2X_2 \leq 6$$
$$2X_1 + X_2 \leq 8$$
$$X_1 \geq 7$$

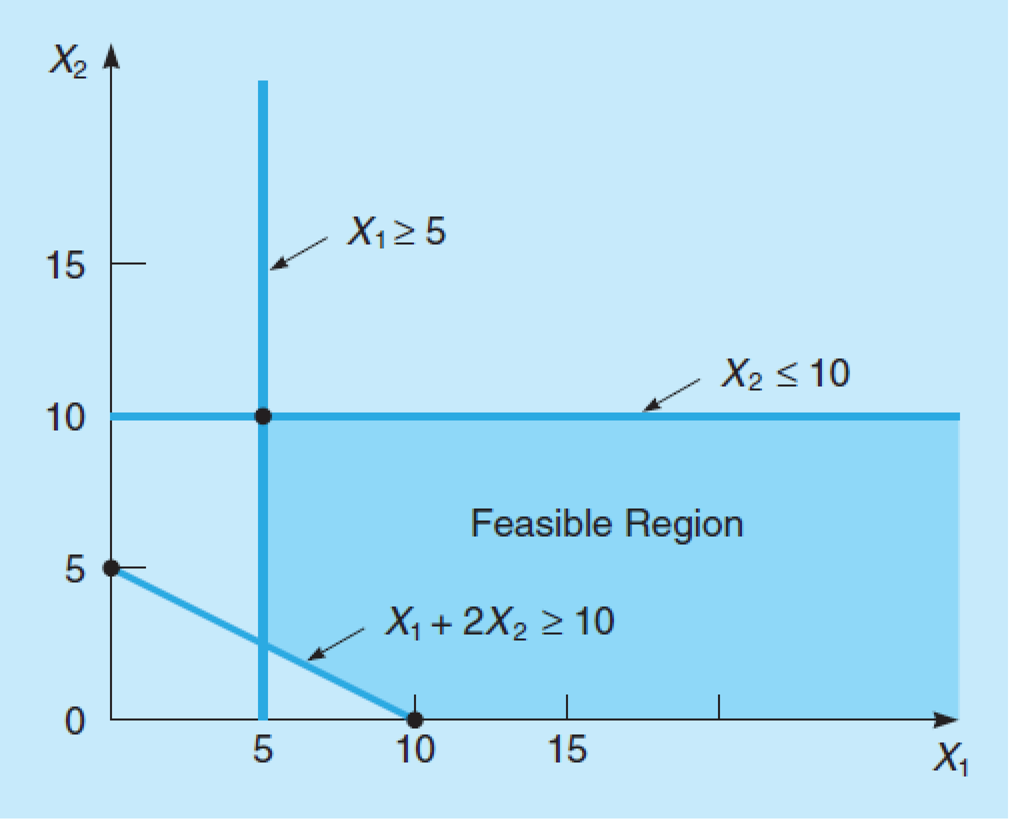


Unboundedness

$$\text{Maximize profit} = \$3X_1 + \$5X_2$$

subject to

$$X_1 \geq 5$$
$$X_2 \leq 10$$
$$X_1 + 2X_2 \geq 10$$
$$X_1, X_2 \geq 0$$

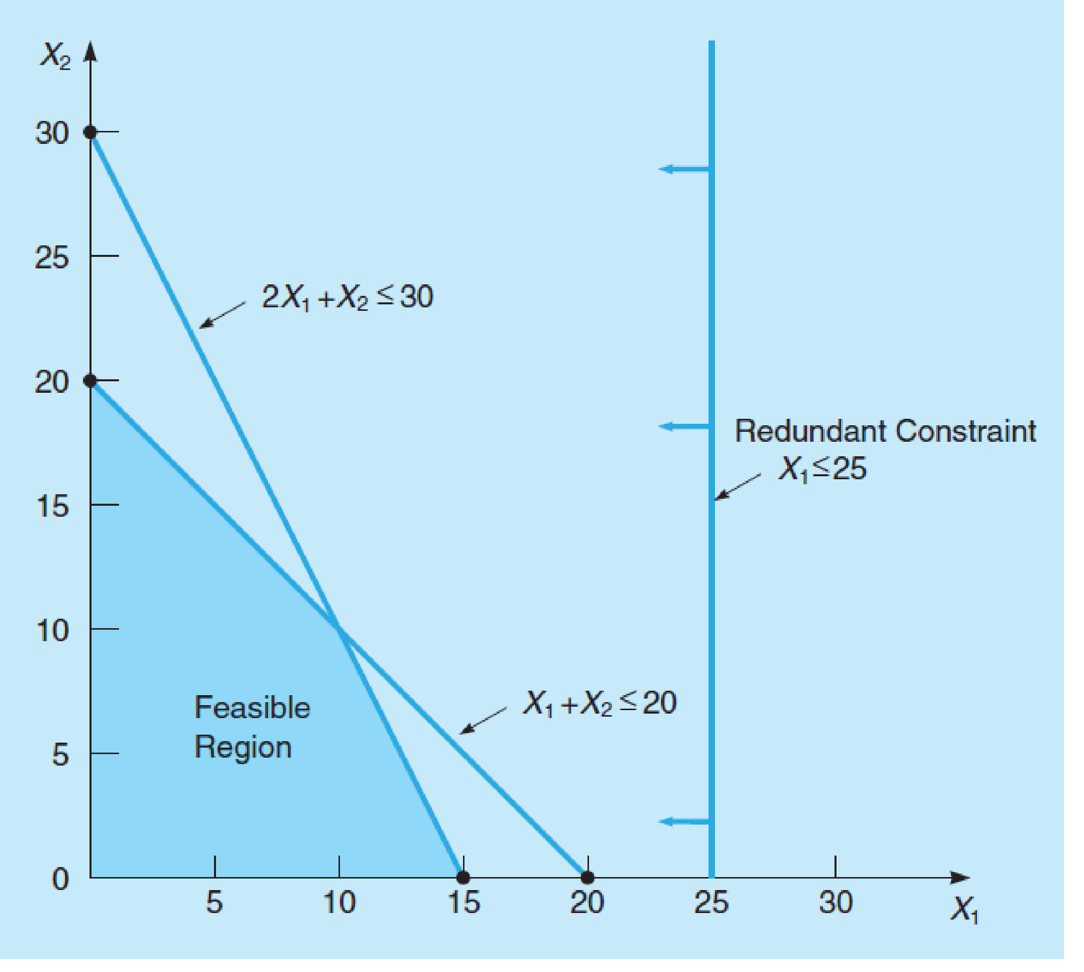


Redundancy

$$\text{Maximize profit} = \$1X_1 + \$2X_2$$

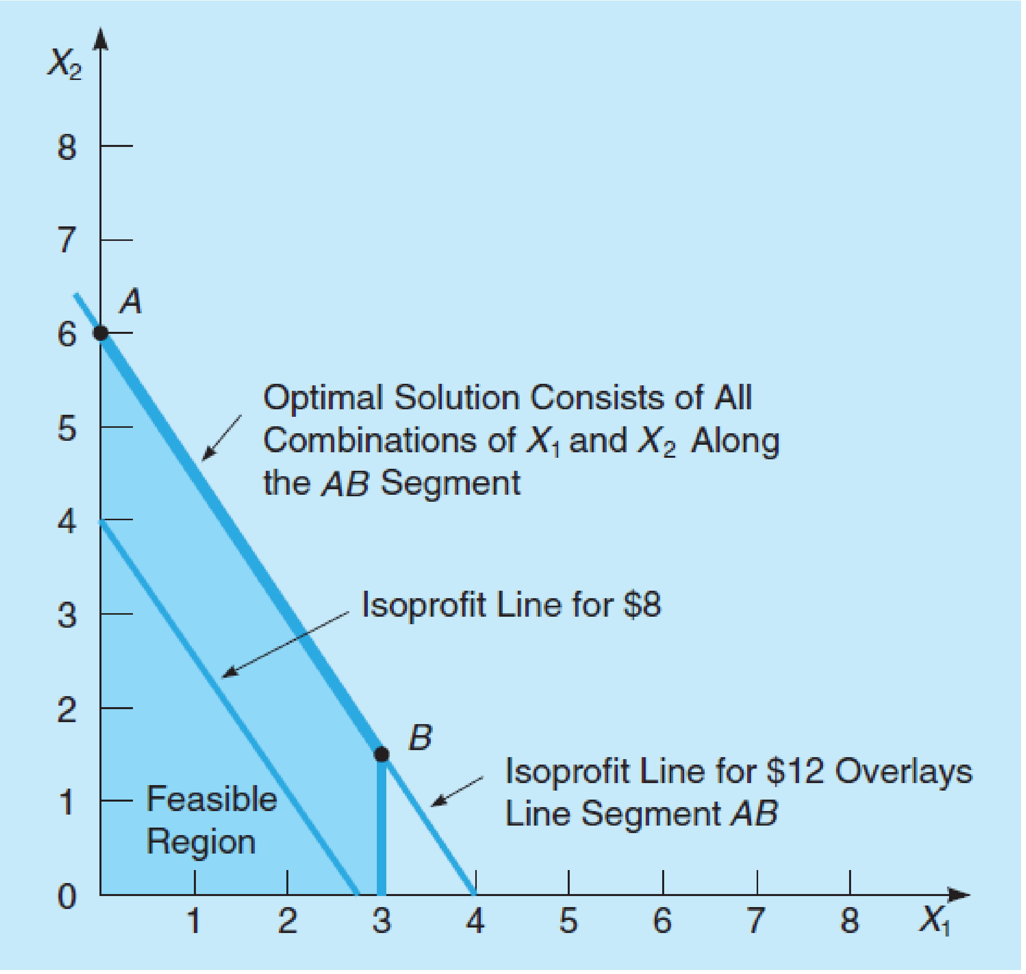
subject to

$$X_1 + X_2 \leq 20$$
$$2X_1 + X_2 \leq 30$$
$$X_1 \leq 25$$
$$X_1, X_2 \geq 0$$



Alternate Optimal Solutions

$$\begin{aligned} \text{Maximize profit} &= \$3X_1 + \$2X_2 \\ \text{subject to} \quad 6X_1 + 4X_2 &\le 24 \\ X_1 &\le 3 \\ X_1, X_2 &\ge 0 \end{aligned}$$



Sensitivity Analysis

How sensitive an optimal solution is to changes

- Trial-and-error approach
- Analytic post-optimality method

EXERCISE

7-14 (cont’d)

The Electrocomp Corporation manufactures two electrical products: air conditioners and large fans. The assembly process for each is similar in that both require a certain amount of wiring and drilling. Each air conditioner takes 3 hours of wiring and 2 hours of drilling. Each fan must go through 2 hours of wiring and 1 hour of drilling.

7-14

During the next production period, 240 hours of wiring time are available and up to 140 hours of drilling time may be used. Each air conditioner sold yields a profit of $25. Each fan assembled may be sold for a $15 profit. Formulate and solve this LP production mix situation to find the best combination of air conditioners and fans that yields the highest profit. Use the corner point graphical approach.

7-16 (cont’d)

A candidate for mayor in a small town has allocated $40,000 for last-minute advertising in the days preceding the election. Two types of ads will be used: radio and television. Each radio ad costs $200 and reaches an estimated 3,000 people. Each television ad costs $500 and reaches an estimated 7,000 people.

7-16

In planning the advertising campaign, the campaign manager would like to reach as many people as possible, but she has stipulated that at least 10 ads of each type must be used. Also, the number of radio ads must be at least as great as the number of television ads. How many ads of each type should be used? How many people will this reach?

7-18

The dean of the Western College of Business must plan the school's course offerings for the fall semester. Student demands make it necessary to offer at least 30 undergraduate and 20 graduate courses in the term. Faculty contracts also dictate that at least 60 courses be offered in total. Each undergraduate course taught costs the college an average of $2,500 in faculty wages, and each graduate course costs $3,000. How many undergraduate and graduate courses should be taught in the fall so that total faculty salaries are kept to a minimum?

7-25 (cont'd)

Woofer Pet Foods produces a low-calorie dog food for overweight dogs. This product is made from beef products and grain. Each pound of beef costs $0.90, and each pound of grain costs $0.60. A pound of the dog food must contain at least 9 units of Vitamin 1 and 10 units of Vitamin 2. A pound of beef contains 10 units of Vitamin 1 and 12 units of Vitamin 2. A pound of grain contains 6 units of Vitamin 1 and 9 units of Vitamin 2.

7-25

Formulate this as an LP problem to minimize the cost of the dog food. How many pounds of beef and grain should be included in each pound of dog food? What is the cost and vitamin content of the final product?

7-26 (cont'd)

The seasonal yield of olives in a Piraeus, Greece, vineyard is greatly influenced by a process of branch pruning. If olive trees are pruned every two weeks, output is increased. The pruning process, however, requires considerably more labor than permitting the olives to grow on their own and results in a smaller size olive. It also, though, permits olive trees to be spaced closer together..

7-26 (cont'd)

The yield of 1 barrel of olives by pruning requires 5 hours of labor and 1 acre of land. The production of a barrel of olives by the normal process requires only 2 labor hours but takes 2 acres of land. An olive grower has 250 hours of labor available and a total of 150 acres for growing. Because of the olive size difference, a barrel of olives produced on pruned trees sells for $20, whereas a barrel of regular olives has a market price of $30.

7-26

The grower has determined that because of uncertain demand, no more than 40 barrels of pruned olives should be produced.

Use graphical LP to find

a) the maximum possible profit.
b) the best combination of barrels of pruned and regular olives.
c) the number of acres that the olive grower should devote to each growing process.

THANK YOU