Decision Making Theory

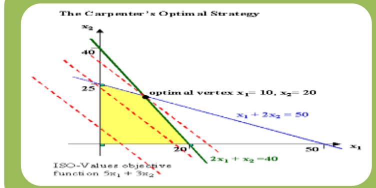
Week 3

Linear Programming

Simplex Method - Maximize

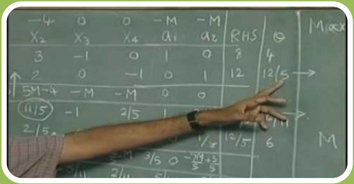
Outlines

Linear Programing Model – Problem Formulation



Linear Programing Model – Graphic Solution

Linear Programing Model – Simplex Method (Maximize)



Linear Programing Model – Simplex Method (Minimize and Non-Standard)

Linear Programming Problems

Standard Maximization

- Objective: Maximize
- Constraints: ALL with ≤
- Standard simplex

Standard Minimization

- Objective: Minimize
- Constraints: All with ≥
- Dual

Non-Standard

- Objective: Max or Min
- Constraints: Mixed, can be ≤, ≥ or =
- Two Phase

STANDARD MAXIMIZATION PROBLEM

Flair Furniture Company

- Objective function:

  Maximize profit (P) = $70T + $50C

- Constraints:

  $4T + 3C \leq 240$

  $2T + 1C \leq 100$

  $T \geq 0$

  $C \geq 0$

Introduce slack variables into constraints to obtain equality (equations)

Slack Variables

All the inequality transformed into equality by adding slack variables to each of the inequality.

$4T + 3C \leq 240$ → $4T + 3C + S_1 = 240$

$2T + 1C \leq 100$ → $2T + 1C + S_2 = 100$

Move the objective function to the left hand side of the equation

$P = \$70T + \$50C$ → $-70T - 50C + P = 0$

First Simplex Tableu

$4T + 3C + S_1 = 240 \rightarrow$ row 1

Basic Variable	T	C	S_1	S_2	P	RHS
S_1	4	3	1	0	0	240
S_2	2	1	0	1	0	100
P	-70	-50	0	0	1	0

$2T + 1C + S_2 = 100 \rightarrow$ row 2

$-70T - 50C + P = 0 \rightarrow$ row 3

Simplex Tableu

Change the pivot number to 1 and other number in the same column to 0

Pivot Number

Basic Variable	T	C	S_1	S_2	P	RHS
S_1	4	3	1	0	0	240
S_2	2	1	0	1	0	100
P	-70	-50	0	0	1	0

= 240/4 = 60

= 100/2 = 50

Most negative, therefore variable T enter the solution mix

Smallest result of the ratio, therefore variable S_2 leave the solution mix.

Second Simplex Tableu

Basic Var.	T	C	S_1	S_2	P	RHS
S_1	4	3	1	0	0	240
S_2	2	1	0	1	0	100
P	-70	-50	0	0	1	0

Basic Variable	T	C	S_1	S_2	P	RHS
S_1						
T						
P						

S_2 leaving the solution mix, while T enter it

Second Simplex Tableu

Basic Var.	T	C	S_1	S_2	P	RHS
S_1	4	3	1	0	0	240
S_2	2	1	0	1	0	100
P	-70	-50	0	0	1	0

The value in pivot number need to be changed into 1

c Variable	T	C	S_1	S_2	P	RHS
S_1						
T	1					
P						

Second Simplex Tableu

Basic Var.	T	C	S_1	S_2	P	RHS
S_1	4	3	1	0	0	240
S_2	2	1	0	1	0	100
P	-70	-50	0	0	1	0

Therefore, all number in row 2 need to be divided by 2

The value in pivot number need to be changed into 1

c Variable	T	C	S_1	S_2	P	RHS
S_1						
T	1					
P						

Second Simplex Tableu

Basic Var.	T	C	S_1	S_2	P	RHS
S_1	4	3	1	0	0	240
S_2	2	1	0	1	0	100
P	-70	-50	0	0	1	0

= 100 : 2
= 50

Variable	T	C	S_1	S_2	P	RHS
S_1						
T	1	1/2		1/2	0	50
P						

= 2 : 2
= 1

= 1 : 2
= 1/2

Second Simplex Tableu

Basic Var.	T	C	S_1	S_2	P	RHS
S_1	4	3	1	0	0	240
S_2	2	1	0	1	0	100
P	-70	-50	0	0	1	0

ariable	T	C	S_1	S_2	P	RHS
S_1	0					
T	1	1/2	0	1/2	0	50
P	0					

Other numbers in the pivot column need to be changed into 0

Second Simplex Tableu

Basic Var.	T	C	S_1	S_2	P	RHS
S_1	4	3	1	0	0	240
S_2	2	1	0	1	0	100
P	-70	-50	0	0	1	0

Use elimination procedure using **row 2** to obtain that number (0)

Other numbers in the pivot column need to be changed into 0

Variable	T	C	S_1	S_2	P	RHS
S_1	0					
T	1	1/2	0	1/2	0	50
P	0					

Second Simplex Tableau

- S_1:

4	3	1	0	0	|	240
2	1	0	1	0	|	100 (x2)
4	3	1	0	0	|	240
4	2	0	2	0	|	200 −
0	1	1	-2	0	|	40

Second Simplex Tableau

Basic Var.	**T**	**C**	**S_1**	**S_2**	**P**	**RHS**
S_1	4	3	1	0	0	240
S_2	2	1	0	1	0	100
P	-70	-50	0	0	1	0

Basic Variable	**T**	**C**	**S_1**	**S_2**	**P**	**RHS**
S_1	**0**	**1**	**1**	**-2**	**0**	**40**
T	1	1/2	0	1/2	0	50
	0					

Do the same calculation for the rest of row 3

Second Simplex Tableu

- P:

-70	-50	0	0	1	|	0
2	1	0	1	0	|	100 (x35)
-70	-50	0	0	1	|	0
70	35	0	35	0	|	3500 +
0	-15	0	35	1	|	3500

Second Simplex Tableu

Basic Var.	T	C	S_1	S_2	P	RHS
S_1	4	3	1	0	0	240
S_2	2	1	0	1	0	100
P	-70	-50	0	0	1	0

Basic Variable	T	C	S_1	S_2	P	RHS
S_1	0	1	1	-2	0	40
T	1	1/2	0	1/2	0	50
P	0	-15	0	35	1	3500

Second Simplex Tableu

Basic Var.	T	C	S_1	S_2	P	RHS	
S_1	4	3	1	0	0	240	= Row 2 * (-2) + Row 1
S_2	2	1	0	1	0	100	= Row 2 / 2
P	-70	-50	0	0	1	0	= Row 2 * (35) + Row 3

Basic Variable	T	C	S_1	S_2	P	RHS
S_1	0	1	1	-2	0	40
T	1	1/2	0	1/2	0	50
P	0	-15	0	35	1	3500

Row P still contain negative value

Second Simplex Tableu

Change the pivot number to 1 and other number in the same column to 0

Pivot Number

Basic Variable	T	C	S_1	S_2	P	RHS
S_1	0	1	1	-2	0	40
T	1	1/2	0	1/2	0	50
P	0	-15	0	35	1	3500

40/1 = 40

50/(1/2) = 100

Most negative, therefore variable C enter the solution mix

Smallest result of the ratio, therefore variable S_1 leave the solution mix

Third Simplex Tableu

Basic Var.	T	C	S_1	S_2	P	RHS	
S_1	0	1	1	-2	0	40	= Row 1
T	1	1/2	0	1/2	0	50	= Row 1 * (-1/2) + Row 2
P	0	-15	0	35	1	3500	= Row 1 * 15 + Row 3

Basic Variable	T	C	S_1	S_2	P	RHS
C		1				
T		0				
P		0				

Use the same procedure as before

Third Simplex Tableu

Basic Var.	T	C	S_1	S_2	P	RHS	
S_1	0	1	1	-2	0	40	= Row 1
T	1	1/2	0	1/2	0	50	= Row 1 * (-1/2) + Row 2
P	0	-15	0	35	1	3500	= Row 1 * 15 + Row 3

Basic Variable	T	C	S_1	S_2	P	RHS
C	0	1	1	-2	0	40
T	1	0	-1/2	3/2	0	30
P	0	0	15	5	1	4100

Third Simplex Tableu

Basic Var.	T	C	S_1	S_2	P	RHS
S_1	0	1	1	-2	0	40
T	1	1/2	0	1/2	0	50
P	0	-15	0	35	1	3500

= Row 1

= Row 1 * (-1/2) + Row 2

= Row 1 * 15 + Row 3

Variable	T	C	S_1	S_2	P	RHS
C	0	1	1	-2	0	40
T	1	0	-1/2	3/2	0	30
	0	0	15	5	1	4100

Note that all the values in row 3 now don't have negative numbers

THEREFORE, OPTIMAL!!

Optimal Solution

C (Chairs) = 40
T (Tables)= 30

Basic Variable	T	C	S_1	S_2	P	RHS
C	0	1	1	-2	0	40
T	1	0	-1/2	3/2	0	30
P	0	0	15	5	1	4100

P (Profit) = 4100

EXERCISE

M7-20

Solve the following LP problem graphically. Then set up a simplex tableau and solve the problem using the simplex method. Indicate the corner points generate at each iteration by the simplex method on your graph.

Maximize profit = $\$3X_1 + \$5X_2$

Subject to $X_2 \leq 6$

$$3X_1 + 2X_2 \leq 18$$

$$X_1, X_2 \geq 0$$

M7-24

Solve the following problem by the simplex method. What condition exists that prevents you from reaching an optimal solution?

Maximize profit = $6X_1 + 3X_2$

Subject to $2X_1 - 2X_2 \leq 2$

$-X_1 + X_2 \leq 1$

$X_1, X_2 \geq 0$

THANK YOU