

Decision Making Theory

Week 4

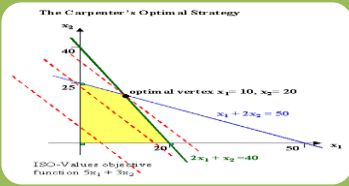
Linear Programming

Simplex Method - Minimize

Outlines



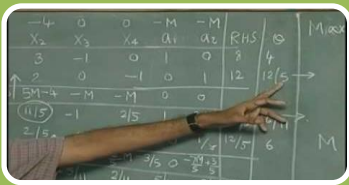
Linear Programming Model – Problem Formulation



Linear Programming Model – Graphic Solution



Linear Programming Model – Simplex Method (Maximize)



Linear Programming Model – Simplex Method (Minimize and Non-Standard)

Linear Programming Problems

Standard Maximization

Objective:
Maximize

Constraints: ALL
with \leq

Standard simplex

Standard Minimization

Objective:
Minimize

Constraints: All
with \geq

Dual

Non-Standard

Objective:
Max or Min

Constraints: Mixed,
can be \leq , \geq or $=$

Two Phase

STANDARD MINIMIZATION PROBLEM

Holiday Meal Turkey Ranch

Minimize cost = $2X_1 + 3X_2$

subject to $5X_1 + 10X_2 \geq 90$

$4X_1 + 3X_2 \geq 48$

$0.5 X_1 \geq 1.5$

$X_1, X_2 \geq 0$

The Dual

$$\text{Minimize cost} = 2X_1 + 3X_2$$

subject to

$$5X_1 + 10X_2 \geq 90$$

$$4X_1 + 3X_2 \geq 48$$

$$0.5 X_1 \geq 1.5$$

$$X_1, X_2 \geq 0$$

$$A = \left[\begin{array}{cc|c} 5 & 10 & 90 \\ 4 & 3 & 48 \\ 0.5 & 0 & 1.5 \\ \hline 2 & 3 & 0 \end{array} \right]$$

$$A^T = \left[\begin{array}{ccc|c} 5 & 4 & 0.5 & 2 \\ 10 & 3 & 0 & 3 \\ \hline 90 & 48 & 1.5 & 0 \end{array} \right]$$

The Dual

$$A^T = \left[\begin{array}{ccc|c} 5 & 4 & 0.5 & 2 \\ 10 & 3 & 0 & 3 \\ \hline 90 & 48 & 1.5 & 0 \end{array} \right]$$

Maximize

$$Z = 90Y_1 + 48Y_2 + 1.5Y_3$$

subject to

$$5Y_1 + 4Y_2 + 0.5Y_3 \leq 2$$

$$10Y_1 + 3Y_2 \leq 3$$

$$Y_1, Y_2, Y_3 \geq 0$$

Continue with the simplex method to
solve standard maximization
problems!!

Slack Variables

When using the dual, slack variables are variables in the former problem formulation. Therefore, slack variables in this problem are X_1 and X_2 .

Constraints:

$$\begin{array}{rcl} 5Y_1 + 4Y_2 + 0.5Y_3 \leq 2 & \longrightarrow & 5Y_1 + 4Y_2 + 0.5Y_3 + X_1 \leq 2 \\ 10Y_1 + 3Y_2 \leq 3 & & 10Y_1 + 3Y_2 + X_2 \leq 3 \end{array}$$


$$Z = 90Y_1 + 48Y_2 + 1.5Y_3$$

$$-90Y_1 - 48Y_2 - 1.5Y_3 + Z = 0$$

First Simplex Tableau

$$5Y_1 + 4Y_2 + 0.5Y_3 + X_1 \leq 2 \rightarrow \text{row 1}$$

Basic Variable	Y_1	Y_2	Y_3	X_1	X_2	Z	RHS
X_1	5	4	0.5	1	0	0	2
X_2	10	3	0	0	1	0	3
Z	-90	-48	-1.5	0	0	1	0



$$10Y_1 + 3Y_2 + X_2 \leq 3 \rightarrow \text{row 2}$$

$$-90Y_1 - 48Y_2 - 1.5Y_3 + Z = 0 \rightarrow \text{row 3}$$

Simplex Tableau

Change the pivot number to 1 and other number in the same column to 0

Pivot Number

Basic Variable	Y_1	Y_2	Y_3	X_1	X_2	Z	RHS
X_1	5	4	0.5	1	0	0	2
X_2	10	3	0	0	1	0	3
Z	-90	-48	-1.5	0	0	1	0

$= 2/5 = 0.4$

$= 3/10 = 0.3$

Most negative, therefore variable Y_1 enter the solution mix

Smallest result of the ratio, therefore variable X_2 leave the solution mix

Second Simplex Tableau

Basic Var.	Y_1	Y_2	Y_3	X_1	X_2	Z	RHS
X_1	5	4	0.5	1	0	0	2 = Row 2 * (-5/10) + Row 1
X_2	10	3	0	0	1	0	3 = Row 2 / 10
Z	-90	-48	-1.5	0	0	1	0 = Row 2 * (90/10) + Row 3



Basic Var.	Y_1	Y_2	Y_3	X_1	X_2	Z	RHS
X_1	0	2.5	0.5	1	-1	0	0.5
Y_1	1	0.3	0	0	0.1	0	0.3
Z	0	-21	-1.5	0	9	1	27

Row P still contain negative value

Second Simplex Tableau

Change the pivot number to 1 and other number in the same column to 0

Pivot Number

Basic Var.	Y ₁	Y ₂	Y ₃	X ₁	X ₂	Z	RHS
X ₁	0	2.5	0.5	1	-1	0	0.5
Y ₁	1	0.3	0	0	0.1	0	0.3
Z	0	-21	-1.5	0	9	1	27

$$0.5/2.5 = 0.2$$

$$0.3/0.3 = 1$$

Most negative, therefore variable Y₂ enter the solution mix

Smallest result of the ratio, therefore variable X₁ leave the solution mix

Third Simplex Tableau

Basic Var.	Y_1	Y_2	Y_3	X_1	X_2	Z	RHS
X_1	0	2.5	0.5	1	-1	0	0.5 = Row 1 / 2.5
Y_1	1	0.3	0	0	0.1	0	0.3 = Row 1 * (-0.3/2.5) + Row 2
Z	0	-21	-1.5	0	9	1	27 = Row 1 * (21/2.5) + Row 3



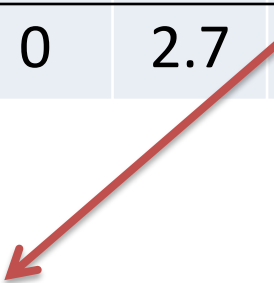
Basic Var.	Y_1	Y_2	Y_3	X_1	X_2	Z	RHS
Y_2	0	1	0.2	0.4	-0.2	0	0.2
Y_1	1	0	-0.06	-0.12	0.16	0	0.24
Z	0	0	2.7	8.4	4.8	1	31.2

OPTIMAL!!

Optimal Solution

Because this solution is the solution of the dual, use row Z and column X_1 , X_2 as the optimal solution.

Basic Var.	Y_1	Y_2	Y_3	X_1	X_2	Z	RHS
Y_2	0	1	0.2	0.4	-0.2	0	0.2
Y_1	1	0	-0.06	-0.12	0.16	0	0.24
Z	0	0	2.7	8.4	4.8	1	31.2


$$X_1 = 8.4$$

$$X_2 = 4.8$$


$$\text{Minimal cost} = 31.2$$

EXERCISE

M7-22

Selesaikan permasalahan program linear berikut dengan menggunakan metode simpleks:

$$\text{Minimasi Biaya} = 4X_1 + 5X_2$$

$$\text{Hambatan } X_1 + 2X_2 \geq 80$$

$$3X_1 + X_2 \geq 75$$

$$X_1, X_2 \geq 0$$

2

Fungsi objektif:

$$\text{Minimasi } w = 3x_1 + 3x_2$$

Hambatan:

$$2x_1 + x_2 \geq 4$$

$$x_1 + 2x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

NON-STANDARD LINEAR PROGRAMMING PROBLEM

The Muddy River Chemical Company

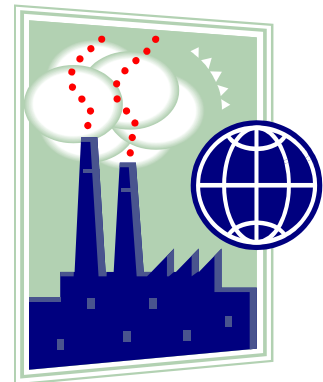
Minimize cost = $\$5X_1 + \$6X_2$

subject to $X_1 + X_2 = 1000$ lb

$X_1 \leq 300$ lb

$X_2 \geq 150$ lb

$X_1, X_2 \geq 0$



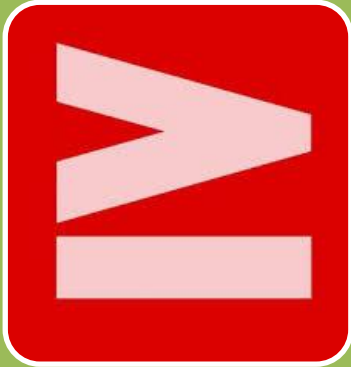
Surplus Variable

Greater-than-or-equal-to (\geq) constraints involve the subtraction of a ***surplus variable*** rather than the addition of a slack variable.

Surplus is sometimes simply called
negative slack

$$X_2 \geq 150 \quad \rightarrow \quad X_2 - S_2 = 150$$

Artificial Variable



Artificial variables are usually added to **greater-than-or-equal-to** (\geq) constraints to avoid violating the non-negativity constraint.



When a constraint is already an **equality** (=), artificial variable is also used to provide an automatic initial solution

Artificial Variable

- Equality

$$X_1 + X_2 = 1000$$



- Equality

$$X_1 + X_2 + A_1 = 1000$$

- Greater than or equal

$$X_2 \geq 150$$



- Greater than or equal

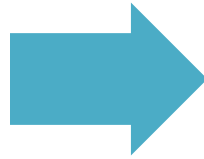
$$X_2 - S_2 + A_2 = 150$$

Before the final simplex solution has been reached, all artificial variables must be gone from the solution mix

Two Phase Method

Phase I

- **Objective:**
Minimize
all
artificial
variables



Phase II

- **Objective:**
LP original
objective

Conversions

Constraints:

$$X_1 + X_2 = 1000$$

$$X_1 \leq 300$$

$$X_2 \geq 150$$

$$X_1 + X_2 + A_1 = 1000$$

$$X_1 + S_1 = 300$$

$$X_2 - S_2 + A_2 = 150$$

Objective function:

$$\text{Minimize } C = 5X_1 + 6X_2$$

Phase I :

$$\text{Minimize } Z = A_1 + A_2$$

Phase II:

$$\text{Minimize } C = 5X_1 + 6X_2$$

Converting The Objective Function

In minimization problem, minimizing the cost objective is the same as ***maximizing the negative of the cost*** objective function

$$\text{Minimize } C = 5X_1 + 6X_2 \text{ and } Z = A_1 + A_2$$



$$\text{Maximize } (-C) = -5X_1 - 6X_2 \text{ and } (-Z) = -A_1 - A_2$$

Conversions

Constraints:

$$X_1 + X_2 = 1000$$

$$X_1 \leq 300$$

$$X_2 \geq 150$$

$$X_1 + X_2 + A_1 = 1000$$

$$X_1 + S_1 = 300$$

$$X_2 - S_2 + A_2 = 150$$

Objective function:

$$\text{Phase I : } (-Z) = -A_1 - A_2$$

Phase II:

$$(-C) = -5X_1 - 6X_2$$

$$\text{Phase I : } A_1 + A_2 - Z = 0$$

Phase II:

$$5X_1 + 6X_2 - C = 0$$

Simplex Tableau

$$X_1 + X_2 + A_1 = 1000 \rightarrow \text{row 1}$$

$$X_1 + S_1 \leq 300 \rightarrow \text{row 2}$$

$$X_2 - S_2 + A_2 \geq 150 \rightarrow \text{row 3}$$

Basic Var.	X_1	X_2	S_1	S_2	A_1	A_2	Z	C	RHS
A_1	1	1	0	0	1	0	0	0	1000
S_1	1	0	1	0	0	0	0	0	300
A_2	0	1	0	-1	0	1	0	0	150
Z	0	0	0	0	1	1	(-1)	0	0
C	5	6	0	0	0	0	0	(-1)	0

$$A_1 + A_2 - Z = 0 \rightarrow \text{row 4}$$

$$5X_1 + 6X_2 - C = 0 \rightarrow \text{row 5}$$

Simplex Tableau

A1 and A2 are supposed to be basic variable. However, in this tableau the condition is not satisfy. → infeasible

Basic Var.	X ₁	X ₂	S ₁	S ₂	A ₁	A ₂	Z	C	RHS
A ₁	1	1	0	0	1	0	0	0	1000
S ₁	1	0	1	0	0	0	0	0	300
A ₂	0	1	0	-1	0	1	0	0	150
Z	0	0	0	0	1	1	(-1)	0	0
C	5	6	0	0	0	0	0	(-1)	0

row 1 * (-1) + row 3 * (-1) + row 4

These values need to be converted to zero (0)

Phase I: First Simplex Tableau

Basic Variable condition is satisfied

Basic Var.	X_1	X_2	S_1	S_2	A_1	A_2	Z	C	RHS
A_1	1	1	0	0	1	0	0	0	1000
S_1	1	0	1	0	0	0	0	0	300
A_2	0	1	0	-1	0	1	0	0	150
Z	-1	-2	0	1	0	0	(-1)	0	-1150
C	5	6	0	0	0	0	0	(-1)	0

Continue with simplex procedure to eliminate the negative values in row Z first (Phase I)

Phase I: Second Simplex Tableau

Change the pivot number to 1 and other number in the same column to 0

Pivot Number

Basic Var.	X_1	X_2	S_1	S_2	A_1	A_2	Z	C	RHS
A_1	1	1	0	0	1	0	$1000/1 = 1000$		1000
S_1	1	0	1	0	0	0	$300/0 = \sim$		300
A_2	0	1	0	-1	0	1	$150/1 = 150$		150
Z	-1	-2	0	1	0	0	(-1)	0	-1150
C	5	6	0	0	0	0	0	(-1)	0

Most negative, therefore variable X_2 enter the solution mix

Smallest result of the ratio, therefore variable A_2 leave the solution mix

Phase I: Second Simplex Tableau

Basic Var.	X_1	X_2	S_1	S_2	A_1	A_2	Z	C	RHS
A_1	1	1	0	0	1				1000
S_1	1	0	1	0	0				300
A_2	0	1	0	-1	0				150
Z	-1	-2	0	1	0				1150
C	5	6	0	0	0				0

$= \text{row 3} * (-1) + \text{row 1}$
 $= \text{row 2}$
 $= \text{row 3}$
 $= \text{row 3} * 2 + \text{row 4}$
 $= \text{row 3} * (-6) + \text{row 5}$



Basic Var.	X_1	X_2	S_1	S_2	A_1	A_2	Z	C	RHS
A_1	1	0	0	1	1	-1	0	0	850
S_1	1	0	1	0	0	0	0	0	300
X_2	0	1	0	-1	0	1	0	0	150
Z	-1	0	0	-1	0	2	(-1)	0	-850
C	5	0	0	6	0	-6	0	(-1)	-900

Row Z still contain negative values

Phase I: Third Simplex Tableau

Change the pivot number to 1 and other number in the same column to 0

Pivot Number

Basic Var.	X_1	X_2	S_1	S_2	A_1	A_2	Z	C	RHS
A_1	1	0	0	1	1	-1	$850/1 = 850$		850
S_1	1	0	1	0	0	0	$300/0 = \sim$		300
X_2	0	1	0	-1	0	1	-		150
Z	-1	0	0	-1	0	2	(-1)	0	-850
C	5	0	0	6	0	-6	0	(-1)	-900

Most negative, therefore variable S_2 enter the solution mix

Smallest result of the ratio, therefore variable A_1 leave the solution mix

Phase I: Third Simplex Tableau

Basic Var.	X_1	X_2	S_1	S_2	A_1	A_2	Z	C	RHS
A_1	1	0	0	1	1				850
S_1	1	0	1	0	0				300
X_2	0	1	0	-1	0				150
Z	-1	0	0	-1	0				-850
C	5	0	0	6	0				-900

= row 1
 = row 2
 = row 1 + row 3
 = row 1 + row 4
 = row 1 * (-6) + row 5



Basic Var.	X_1	X_2	S_1	S_2	A_1	A_2	Z	C	RHS
S_2	1	0	0	1	1	-1	0	0	850
S_1	1	0	1	0	0	0	0	0	300
X_2	1	1	0	0	1	0	0	0	1000
Z	0	0	0	0	1	1	(-1)	0	0
C	1	0	0	0	6	0	0	(-1)	-6000

Phase I OPTIMAL!!

Phase II: First Simplex Tableau

Change the pivot number to 1 and other number in the same column to 0

Pivot Number

Basic Var.	X_1	X_2	S_1	S_2	C	RHS
S_2	1	0	0	1	0	850
S_1	1	0	1	0	0	300
X_2	1	1	0	0	0	1000
C	-1	0	0	0	(-1)	-6000

$$850/1 = 850$$

$$300/1 = 300$$

$$1000/1 = 1000$$

Most negative, therefore variable X_1 enter the solution mix

Smallest result of the ratio, therefore variable S_1 leave the solution mix

Third Simplex Tableau

Basic Var.	X_1	X_2	S_1	S_2	C	RHS
S_2	1	0	0	1	0	850
S_1	1	0	1	0	0	300
X_2	1	1	0	0	0	1000
C	-1	0	0	0	(-1)	-6000

= Row 2 * (-1) + row 1

= Row 2

= Row 2 * (-1) + Row 3

= Row 2 + Row 4



Basic Var.	X_1	X_2	S_1	S_2	C	RHS
S_2	0	0	-1	1	0	550
X_1	1	0	1	0	0	300
X_2	0	1	-1	0	0	700
C	0	0	1	0	(-1)	-5700


OPTIMAL!!

Optimal Solution

Remember that the objective function was converted. Thus:

$$\text{Max } (-C) = -5700 \rightarrow \text{Min } C = 5700$$

Basic Var.	X_1	X_2	S_1	S_2	C	RHS
S_2	0	0	-1	1	0	550
X_1	1	0	1	0	0	300
X_2	0	1	-1	0	0	700
C	0	0	1	0	(-1)	-5700


$$X_1 = 300$$
$$X_2 = 700$$

EXERCISE

M7-27

A Pharmaceutical firm is about to begin production of three new drugs. An objective function designed to minimize ingredients costs and three production constraints are as follows:

$$\text{Minimize cost} = 50X_1 + 10X_2 + 75X_3$$

$$\text{subject to } X_1 - X_2 = 1.000$$

$$2X_2 + 2X_3 = 2.000$$

$$X_1 \leq 1.500$$

$$X_1, X_2, X_3 \geq 0$$

SPECIAL CASE IN SIMPLEX METHOD

1. Infeasibility

No feasible solution is possible if an artificial variable remains in the solution mix

Basic Var.	X_1	X_2	S_1	S_2	A_1	A_2	Z	C	RHS
X_1	1	0	-2	3	-1	0	0	0	200
X_2	0	1	1	2	-2	0	0	0	100
A_2	0	0	0	-1	-1	1	0	0	20
Z	0	0	2	3	0	0	(-1)	0	-850
C	0	0	0	-31	21	0	0	(-1)	-900



Phase I already optimal (no negative value)

2. Unbounded Solutions

Unboundedness describes linear programs that do not have finite solutions

Basic Var.	X_1	X_2	S_1	S_2	P	RHS
X_2	-1	1	2	0	0	30
S_2	-2	0	-1	1	0	10
P	-15	0	18	0	1	270

Since both pivot column numbers are negative, an unbounded solution is indicated

3. Degeneracy

Degeneracy develops when three constraints pass through a single point

Basic Var.	X_1	X_2	X_3	S_1	S_2	S_3	C	RHS
X_2	0.25	1	1		$10/0.25 = 40$		0	10
S_2	4	0	0.33		$20/4 = 5$		0	20
S_3	2	0	2		$10/2 = 5$		0	10
C	-3	0	6	16	0	0	1	80

Tie for the smallest ratio indicates degeneracy

Degeneracy could lead to a situation known as **cycling**, in which the simplex algorithm alternates back and forth between the same non-optimal solutions

4. More Than One Optimal Solution

If the value of P (objective function's row) is equal to 0 for a variable that is not in the solution mix, more than one optimal solution exists.

Basic Var.	X_1	X_2	S_1	S_2	P	RHS
X_2	1.5	1	1	0	0	6
S_2	1	0	0.5	1	0	3
P	0	0	2	0	1	12

HOMEWORK

1

John's Locomotive Works memproduksi model lokomotif. Terdapat dua versi untuk produk tersebut— model standar (X_1), dan model deluxe (X_2). Versi standar dapat mendatangkan keuntungan \$250 per lokomotif, sementara versi deluxe mendatangkan keuntungan \$350 per lokomotif. Satu hambatan yang dimiliki John adalah waktu produksi. Ia hanya memiliki 40 jam per minggu untuk perakitan. Versi standar membutuhkan waktu 250 menit untuk masing-masingnya, sementara versi deluxe membutuhkan waktu 350 menit.

1

Mesin penggilingan John juga memiliki hambatan. Hanya terdapat 20 jam per minggu yang tersedia untuk mesin penggilingan. Versi standar membutuhkan 60 menit di mesin penggilingan, sementara versi deluxe membutuhkan 120 menit. Formulasikan permasalahan linear programming tersebut, dan selesaikan dengan menggunakan metode grafis.

2

Queen City Nursery memproduksi media tanam yang terdiri atas *topsoil* dan kompos. Setiap kaki kubik kompos membutuhkan biaya 12 ribu rupiah dan mengandung 4 pon pasir, 3 pon tanah, dan 5 pon humus. Setiap kaki kubik media tanam membutuhkan biaya sebesar 20 ribu dan mengandung 3 pon pasir, 6 pon tanah, dan 12 pon humus. Setiap kantong media tanam tersebut harus mengandung paling tidak 12 pon pasir, 12 pon tanah, dan 10 pon humus.

2

Gambarkanlah hambatan-hambatan pada permasalahan ini dan identifikasikan area *feasible*-nya. Tentukan kombinasi terbaik dari kompos dan topsoil yang memenuhi kondisi yang diinginkan tersebut. Identifikasikan biaya terendah yang mungkin didapatkan.

3

Apakah kombinasi x dan y yang memberikan hasil optimal untuk permasalahan berikut?

Maksimasi $\$3x + \$15y$,

Hambatan

(1) $2x + 4y \leq 12$ dan

(2) $5x + 2y \leq 10$.

4

Apakah kombinasi x dan y yang memberikan hasil optimal untuk permasalahan berikut?

$$\text{Minimasi } C = 2x_1 + x_2$$

Hambatan

$$5x_1 + x_2 \geq 9,$$

$$2x_1 + 2x_2 \leq 10, \text{ dan}$$

$$x_1, x_2 \geq 0$$

THANK YOU