

Decision Making Theory

Week 5 –

Transportation Models

Outline

- ✓ Transportation Modeling
- ✓ Developing an Initial Solution
 - ✓ The Northwest-Corner Rule
 - ✓ The Intuitive Lowest-Cost Method
 - ✓ Vogel's Approximation Method
- ✓ Optimization method
 - ✓ The Stepping-Stone Method
 - ✓ MODI (Modified Distribution) Method
- ✓ Special Issues in Modeling
 - ✓ Demand Not Equal to Supply
 - ✓ Degeneracy

Learning Objectives

When you complete this module you should be able to:

1. Develop an initial solution to a transportation models with the northwest-corner, intuitive lowest-cost methods and Vogel's Approximation Method
2. Solve a problem with the stepping-stone and MODI (Modified Distribution method)
3. Balance a transportation problem
4. Solve a problem with degeneracy

Transportation Modeling

- ✓ An interactive procedure that finds the least costly means of moving products from a series of sources to a series of destinations
- ✓ Can be used to help resolve distribution and location decisions



Transportation Modeling

☑ A special class of linear programming

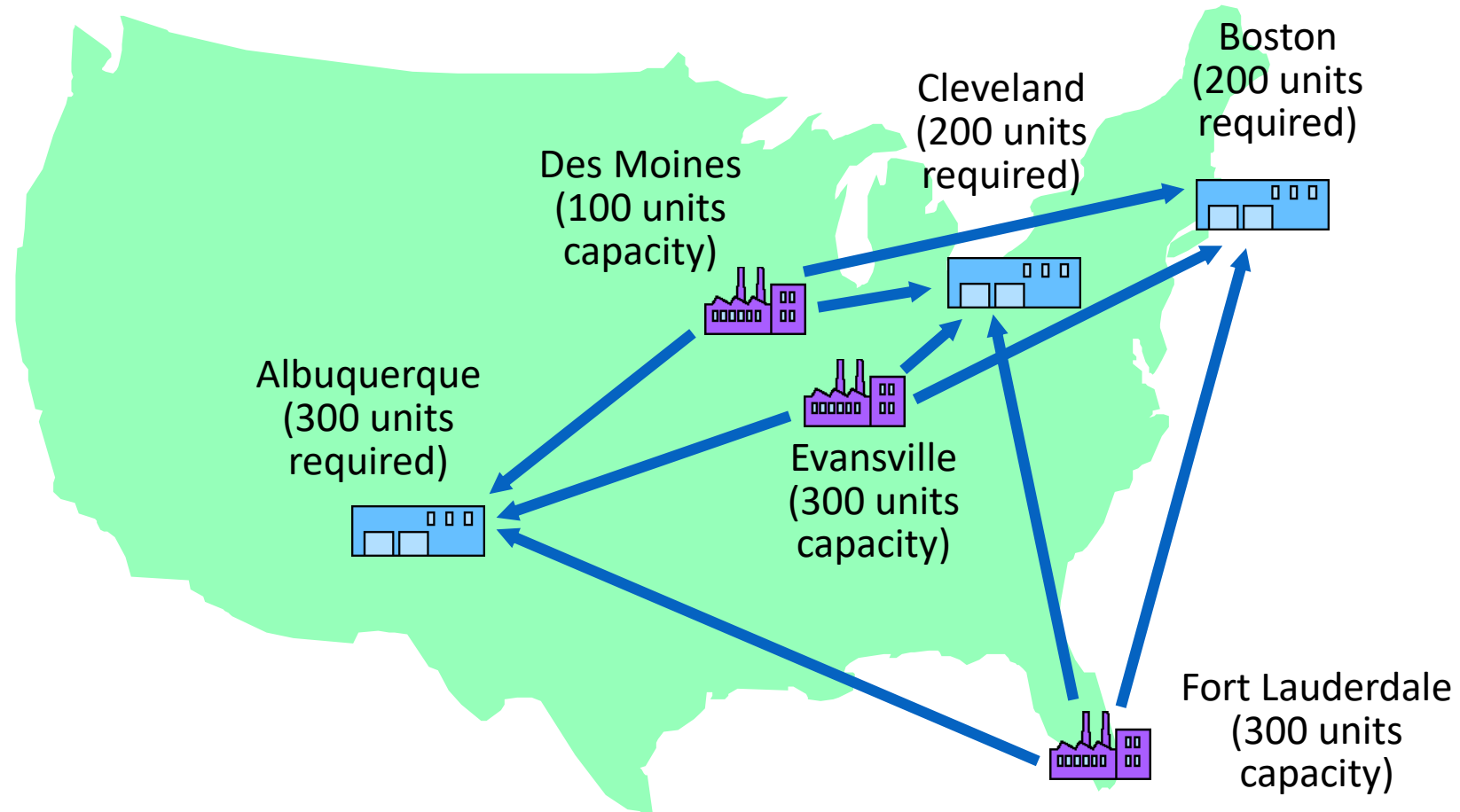
☑ Need to know

1. The origin points and the capacity or supply per period at each
2. The destination points and the demand per period at each
3. The cost of shipping one unit from each origin to each destination

Transportation Problem

<i>From</i>	<i>To</i>	<i>Albuquerque</i>	<i>Boston</i>	<i>Cleveland</i>
<i>Des Moines</i>		\$5	\$4	\$3
<i>Evansville</i>		\$8	\$4	\$3
<i>Fort Lauderdale</i>		\$9	\$7	\$5

Transportation Problem



Transportation Matrix

From \ To	Albuquerque	Boston	Cleveland	Factory capacity
Des Moines	\$5	\$4	\$3	100
Evansville	\$8	\$4	\$3	300
Fort Lauderdale	\$9	\$7	\$5	300
Warehouse requirement	300	200	200	700

Des Moines capacity constraint

Cell representing a possible source-to-destination shipping assignment (Evansville to Cleveland)

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Cleveland warehouse demand

Total demand and total supply

Initial solution

Northwest-Corner Rule

- ☑ Start in the upper left-hand cell (or northwest corner) of the table and allocate units to shipping routes as follows:
 1. Exhaust the supply (factory capacity) of each row before moving down to the next row
 2. Exhaust the (warehouse) requirements of each column before moving to the next column
 3. Check to ensure that all supplies and demands are met

Northwest-Corner Rule

1. Assign 100 tubs from Des Moines to Albuquerque (exhausting Des Moines's supply)
2. Assign 200 tubs from Evansville to Albuquerque (exhausting Albuquerque's demand)
3. Assign 100 tubs from Evansville to Boston (exhausting Evansville's supply)
4. Assign 100 tubs from Fort Lauderdale to Boston (exhausting Boston's demand)
5. Assign 200 tubs from Fort Lauderdale to Cleveland (exhausting Cleveland's demand and Fort Lauderdale's supply)

Northwest-Corner Rule

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 \$5	\$4	\$3	100
(E) Evansville	200 \$8	100 \$4	\$3	300
(F) Fort Lauderdale	\$9	100 \$7	200 \$5	300
Warehouse requirement	300	200	200	700

Means that the firm is shipping 100 bathtubs from Fort Lauderdale to Boston

Northwest-Corner Rule

Computed Shipping Cost

Route		Tubs Shipped	Cost per Unit	Total Cost
From	To			
D	A	100	\$5	\$ 500
E	A	200	8	1,600
E	B	100	4	400
F	B	100	7	700
F	C	200	5	\$1,000
				<u>Total: \$4,200</u>

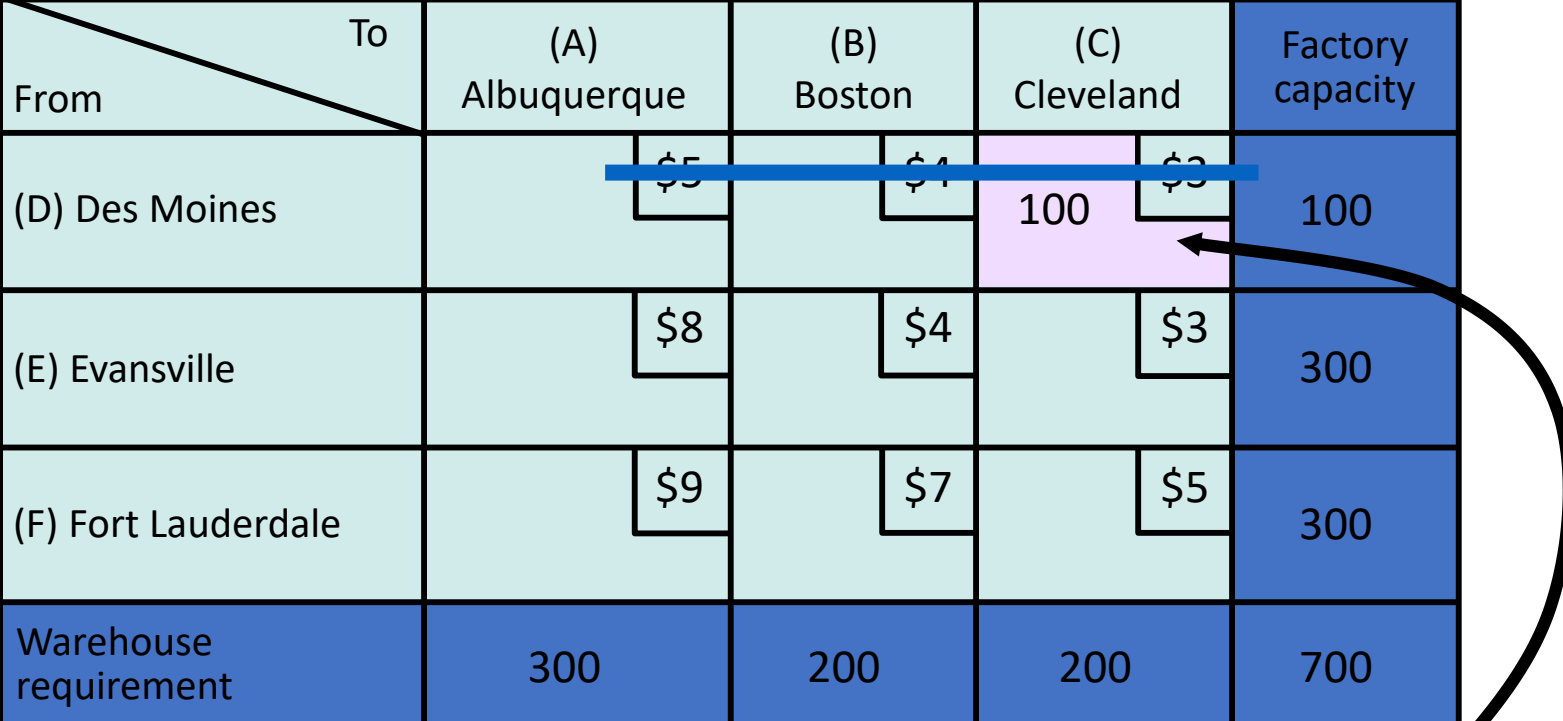
This is a feasible solution but not necessarily the lowest cost alternative

Intuitive Lowest-Cost Method

1. Identify the cell with the lowest cost
2. Allocate as many units as possible to that cell without exceeding supply or demand; then cross out the row or column (or both) that is exhausted by this assignment
3. Find the cell with the lowest cost from the remaining cells
4. Repeat steps 2 and 3 until all units have been allocated

Intuitive Lowest-Cost Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	\$5	\$4	\$3 100	100
(E) Evansville	\$8	\$4	\$3	300
(F) Fort Lauderdale	\$9	\$7	\$5	300
Warehouse requirement	300	200	200	700



First, \$3 is the lowest cost cell so ship 100 units from Des Moines to Cleveland and cross off the first row as Des Moines is satisfied

Intuitive Lowest-Cost Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	\$5	\$4	100	100
(E) Evansville	\$8	\$4	100	300
(F) Fort Lauderdale	\$9	\$7	\$5	300
Warehouse requirement	300	200	200	700

Second, \$3 is again the lowest cost cell so ship 100 units from Evansville to Cleveland and cross off column C as Cleveland is satisfied

Intuitive Lowest-Cost Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	\$5	\$4	\$3	100
(E) Evansville	\$8	\$4	\$3	300
(F) Fort Lauderdale	\$9	\$7	\$5	300
Warehouse requirement	300	200	200	700

Third, \$4 is the lowest cost cell so ship 200 units from Evansville to Boston and cross off column B and row E as Evansville and Boston are satisfied

Intuitive Lowest-Cost Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	\$5	\$4	\$3	100
(E) Evansville	\$8	\$4	\$3	300
(F) Fort Lauderdale	\$9	\$7	\$5	300
Warehouse requirement	300	200	200	700

Finally, ship 300 units from Albuquerque to Fort Lauderdale as this is the only remaining cell to complete the allocations

Intuitive Lowest-Cost Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	\$5	\$4	\$3 100	100
(E) Evansville	\$8	\$4 200	\$3 100	300
(F) Fort Lauderdale	\$9 300	\$7	\$5	300
Warehouse requirement	300	200	200	700

$$\begin{aligned}
 \text{Total Cost} &= \$3(100) + \$3(100) + \$4(200) + \$9(300) \\
 &= \$4,100
 \end{aligned}$$

Intuitive Lowest-Cost Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
		\$4	\$3	100
		\$4	\$3	300
(F) Fort Lauderdale	300 \$9	\$7	\$5	300
Warehouse requirement	300	200	200	700

This is a feasible solution, and an improvement over the previous solution, but not necessarily the lowest cost alternative

$$\begin{aligned} \text{Total Cost} &= \$3(100) + \$3(100) + \$4(200) + \$9(300) \\ &= \$4,100 \end{aligned}$$

Figure C.4

Vogel's Approximation Method

1. For each row and column of the transportation table, find the difference between the two lowest unit shipping costs.
2. Identify the row or column with the greatest opportunity cost, or difference.
3. Assign as many units as possible to the lowest cost square in the row or column selected.
4. Eliminate any row or column that has just been completely satisfied by the assignment just made.
5. Recompute the cost differences for the transportation table.

Vogel's Approximation Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity	
(D) Des Moines	\$5	\$4	\$3	100	4-3=1
(E) Evansville	\$8	\$4	\$3	300	4-3=1
(F) Fort Lauderdale	\$9	\$7	\$5	300	7-5=2
Warehouse requirement	300	200	200	700	

8-5=3
4-4=0 **3-3=0**

Vogel's Approximation Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 \$5	X \$4	X \$3	100
(E) Evansville				300
(F) Fort Lauderdale				300
Warehouse requirement	300	200	200	700

$$4-3=1$$

$$7-5=2$$

$$9-8=1$$

$$7-4=3$$

$$5-3=2$$

Vogel's Approximation Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 \$5	X \$4	X \$3	100
(E) Evansville	X \$8	200 \$4	100 \$3	300
(F) Fort Lauderdale	\$9	X \$7	\$5	300
Warehouse requirement	300	200	200	700

$$8-3=5$$

$$9-5=4$$

$$9-8=1$$

$$5-3=2$$

Vogel's Approximation Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 \$5	X \$4	X \$3	100
(E) Evansville	X \$8	200 \$4	100 \$3	300
(F) Fort Lauderdale	200 \$9	X \$7	100 \$5	300
Warehouse requirement	300	200	200	700

$$\begin{aligned}
 \text{Total Cost} &= \$5(100) + \$4(200) + \$3(100) + \$9(200) + \$5(100) \\
 &= \$3,900
 \end{aligned}$$

exercise

9-16 (1)

The Saussy Lumber Company ships pine flooring to three building supply houses from its mills in Pineville, Oak Ridge, and Mapletown. Determine the best transportation schedule for the data given in the table. Use the northwest corner rule and the stepping-stone method

9-16 (2)

To From	Supply House 1	Supply House 2	Supply House 3	Mill Capacity (Tons)
Pineville	\$3	\$3	\$2	25
Oak Ridge	4	2	3	40
Mapletown	3	2	3	30
Supply House Demand (Tons)	30	30	35	95

9-17 (1)

The Krampf Lines Railway Company specializes in coal handling. On Friday, April 13, Krampf had empty cars at the following towns in the quantities indicated:

TOWN	SUPPLY OF CARS
Morgantown	35
Youngstown	60
Pittsburgh	25

9-17 (2)

By Monday, April 16, the following towns will need coal cars as follows:

TOWN	DEMAND FOR CARS
Coal Valley	30
Coaltown	45
Coal Junction	25
Coalsburg	20

9-17 (3)

Using a railway city-to-city distance chart, the dispatcher constructs a mileage table for the preceding towns. The result is shown in the table below. Minimizing total miles over which cars are moved to new locations, compute the best shipment of coal cars.

FROM \ TO	COAL VALLEY	COALTOWN	COAL JUNCTION	COALSBURG
MORGANTOWN	50	30	60	70
YOUNGSTOWN	20	80	10	90
PITTSBURGH	100	40	80	30

Thank you