

# DECISION MAKING THEORY

Week 11 – Analytical Hierarchy  
Process

# Analytical Hierarchy Process (AHP)

- AHP is a decision approach designed to aid in the solution of **complex multiple criteria problems** in a number of application domain
- AHP is a decision making tool that decomposes **a complex problem into a multilevel hierarchical structure of objective**, criteria, sub-criteria and alternatives
- AHP is designed to cope with **both the rational and the intuitive** to select the best from a number of alternatives evaluated with respect to several criteria

# Analytical Hierarchy Process (AHP)

- In this process, the decision maker carries out **simple pair-wise comparison judgments** (to judge the importance of each criteria) which are then used to develop overall priorities for ranking the alternatives
- AHP allows for inconsistency in the judgments and provides a means to improve consistency

# Analytical Hierarchy Process (AHP)

- Principles the AHP:
  - Decomposition: structuring the elements of the problem into a hierarchy
  - Comparative judgments: generating a matrix of pairwise comparisons of all elements in a level with respect to each related element in the level immediately above it where the principal right eigenvector of the matrix provides ratio scaled priority ratings for the set elements compared
  - Synthesis of priorities: calculating the global or composite priority of the elements at the lowest level of the hierarchy

# Analytical Hierarchy Process (AHP)

- Rationality in AHP is defined as:
  - Focusing on the goal of solving the problem
  - Knowing enough about a problem to develop a thorough structure of relations and influences
  - Having enough knowledge and experience and access to knowledge and experience of others to access the priority of influence and dominance (importance, preference or likelihood to the goal as appropriate) among the relations in the structure
  - Allowing for difference in opinion with an ability to develop a best compromise

# Analytical Hierarchy Process (AHP)

Involving Four Phase :

1. Structuring the decision problem
  - Problem decomposition
  - Define the criteria
  - Design of hierarchy
2. Measuring and collecting data
  - Pair-wise comparison procedure
  - Rate the alternatives

# Analytical Hierarchy Process (AHP)

Involving Four Phase :

3. Determining the normalized weights
  - Calculate the weight of the criteria
  - Calculate the consistency ratio (CR)
4. Synthesis-finding solution to the problem
  - Compute the overall score
  - Overall decision

# Example AHP (1)

## 1. Structuring the decision problem

Level 1:  
Goal

Buy the best  
car

Level 2:  
criteria

Price

MPG

Comfort

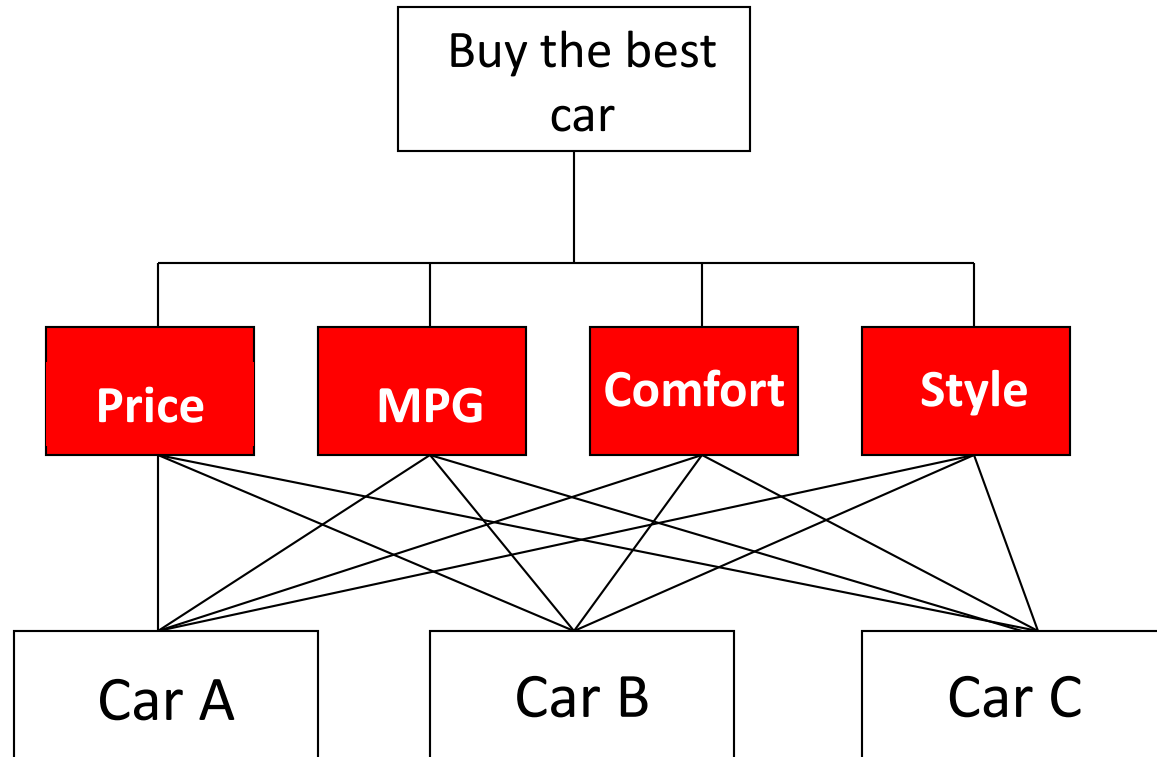
Style

Level 3:  
Alternatives

Car A

Car B

Car C





# Example AHP (2)

## 2. Measuring and collecting data (1)

### Pair-wise Comparison Procedure

- Comparisons ask 2 questions:
  - Which is more important with respect to the criterion?
  - How strongly?
- Matrix shows results of all such comparisons
- Typically use a nine point scale (1-9 scale)
- Requires  $n(n-1)/2$  judgments
- Information and the priority weights of elements may be obtained from a decision maker using direct questioning or questionnaire method
- Inconsistency may arise
- Relationship between two elements that share a common parent in the hierarchy

# Example AHP (3)

## 2. Measuring and collecting data (2)

We use the following for pairwise comparison:

1. —Equally preferred (sama)
2. —Equally to moderately preferred
3. —Moderately preferred (cukup disukai)
4. —Moderately to strongly preferred
5. —Strongly preferred (lebih disukai)
6. —Strongly to very strongly preferred
7. —Very strongly preferred (sangat lebih disukai)
8. —Very to extremely strongly preferred
9. —Extremely preferred

# Example AHP (4)

## 2. Measuring and collecting data (3)

- Establish priorities:
  - The priorities of the four criteria in terms of over all goal
  - The priorities of the three cars in terms of the price criterion
  - The priorities of the three cars in terms of the MPG criterion
  - The priorities of the three cars in terms of the comfort criterion
  - The priorities of the three cars in terms of the style criterion

## Example AHP (5)

### 2. Measuring and collecting data (4)

Criterion	Price	MPG	Comfort	Style
Price	1	3	2	2
MPG		1		
Comfort		4	1	
Style		4	2	1
Total				

Pair-wise Comparison in terms of all four criteria

## Example AHP (5)

### 2. Measuring and collecting data (4)

Criterion	Price	MPG	Comfort	Style
Price	1	3	2	2
MPG	0.33	1	0.25	0.25
Comfort	0.5	4	1	0.5
Style	0.5	4	2	1
Total	2.33	12	5.25	3.75

Pair-wise Comparison in terms of all four criteria

## Example AHP (6)

### 3. Determining the normalized weights (1)

Price	MPG	Comfort	Style	Priority vector
<b>0.43</b>	0.25	0.38	0.53	<b>0.3982</b>
0.14	0.08	0.05	0.07	0.0851
0.21	0.33	0.19	0.13	0.2179
0.21	0.33	0.38	0.27	0.2988

$= 1 / \text{sum of column 1}$   
 $= 1 / (1 + 0.33 + 0.5 + 0.5)$   
 $= 0.43$

Priority vector in terms of all four

$= (0.43 + 0.25 + 0.38 + 0.53) / 4$   
 $= 0.3982$

## Example AHP (7)

### Calculate the Consistency Ratio (1)

#### 1. Define the relative priority

Criterion	Price	MPG	Comfort	Style	Priority vector
Price	0.43	0.25	0.38	0.53	0.40
MPG	0.14	0.08	0.05	0.07	0.08
Comfort	0.21	0.33	0.19	0.13	0.22
Style	0.21	0.33	0.38	0.27	0.30

# Example AHP (8)

## Calculate the Consistency Ratio (2)

### 2. Calculate the Consistency vector

Vector Priority =  $1.69 / 0.40 = 4.23$

Criterion	Price (0.40)	MPG (0.08)	Comfort (0.22)	Style (0.30)
Price	1	3	2	2
MPG	0.33	1	0.25	0.25
Comfort	0.5	4	1	0.5
Style	0.5	4	2	1

$= 1 * 0.4 = 0.4$

$= 4 * 0.08 = 0.34$

Criterion.	Price	MPG	Comfort	Style	Row Sumation	Consistency vector
Price	0.40	0.25	0.44	0.60	1.69	4.23
MPG	0.13	0.08	0.05	0.07	0.35	4.07
Comfort	0.20	0.34	0.22	0.15	0.91	4.16
Style	0.20	0.34	0.44	0.30	1.27	4.26



## Example AHP (9)

### Calculate the Consistency Ratio (3)

#### 3. Calculate the Consistency Index (CI)

$$\lambda = \frac{\text{Consistency vector}_1 + \text{Consistency vector}_1 + \dots + \text{Consistency vector}_n}{n}$$

$$\lambda = \frac{4.23 + 4.07 + 4.16 + 4.26}{4} = 4.18$$

$$CI = \frac{\lambda - n}{n - 1}$$

$$CI = \frac{4.18 - 4}{4 - 3} = 0.06$$

## Example AHP (10)

### Calculate the Consistency Ratio (4)

#### 4. Calculate Consistency Ratio (CR)

$$CR = \frac{CI}{RI}$$

- The acceptable CR range varies according to the size of matrix: 0,10
- If CR is equal to or less than that value, it implies that the evaluation within the matrix is acceptable or indicates a good level of consistency in the comparative judgments represented in that matrix. Otherwise, inconsistency has occurred and the evaluation process should therefore be reviewed, reconsidered and improved

## Example AHP (11)

Calculate the Consistency Ratio (5)

4. Calculate Consistency Ratio (CR)

$$CR = \frac{CI}{RI}$$

Where:

Size of matrix (n)	RI	Size of matrix (n)	RI
1	0	5	1.12
2	0	6	1.24
3	0.58	7	1.32
4	0.90	8	1.41

## Example AHP (12)

Calculate the Consistency Ratio (6)

4. Calculate Consistency Ratio (CR)

$$CR = \frac{CI}{RI}$$

$$CR = \frac{0.06}{0.89} = 0.068 \leq 0.10$$

Therefore, the matrix is consistent

NOTES: Do this calculation for all matrices to check the consistency of the judgment

## Example AHP (13)

### 2. Measuring and collecting data (5)

Price	Car A	Car B	Car C
Car A	1		
Car B	3	1	
Car C	4	2	1

Pair-wise Comparison in terms of **Price**

# Example AHP (15)

## 2. Measuring and collecting data (6)

MPG	Car A	Car B	Car C
Car A	1		
Car B	4	1	
Car C	6	3	1

Pair-wise Comparison in terms of **MPG**

## Example AHP (17)

### 2. Measuring and collecting data (7)

Comfort	Car A	Car B	Car C
Car A	1	2	8
Car B		1	6
Car C			1

Pair-wise Comparison in terms of **Comfort**

# Example AHP (19)

## 2. Measuring and collecting data (8)

Style	Car A	Car B	Car C
Car A	1		4
Car B	3	1	7
Car C			1

Pair-wise Comparison in terms of **Style**



## Example AHP (21)

### 4. Synthesis-finding solution to the problem (1)

Alternative	Price	MPG	Comfort	Style
Car A	0.123	0.087	0.593	0.265
Car B	0.32	0.274	0.341	0.655
Car C	0.557	0.639	0.065	0.08
Overall	0.398	0.085	0.218	0.299

## Example AHP (22)

### 4. Synthesis-finding solution to the problem (2)

- Car A:  $0.398 \times 0.123 + 0.085 \times 0.087 +$   
 $0.0218 \times 0.593 + 0.299 \times 0.265 = 0.265$
- Car B:  $0.398 \times 0.32 + 0.085 \times 0.274 +$   
 $0.0218 \times 0.341 + 0.299 \times 0.655 = 0.421$
- Car C:  $0.398 \times 0.557 + 0.085 \times 0.639 +$   
 $0.0218 \times 0.065 + 0.299 \times 0.08 = 0.314$

Final AHP ranking : B – C – A

**EXERCISE**

# M1-9

Gina Fox is a student who will be graduating soon, and she is planning to attend graduate school to work toward an MBA. Gina has been accepted into the graduate programs at three universities. Now she must decide which one to attend. Therefore, she has decided to compare the universities two at a time (pairwise comparison).

# M1-9

On cost, B is strongly preferred to A; B is moderately preferred to C; and C is moderately preferred to A.

B – 9 8 7 6 (5) 4 3 2 1 2 3 4 5 6 7 8 9 – A

B – 9 8 7 6 5 4 (3) 2 1 2 3 4 5 6 7 8 9 – C

C – 9 8 7 6 5 4 (3) 2 1 2 3 4 5 6 7 8 9 – A

# M1-9

On reputation, A is very strongly preferred to B;  
C is moderately preferred to B; and A is strongly  
preferred to C.

A – 9 8 **7** 6 5 4 3 2 1 2 3 4 5 6 7 8 9 – B

C – 9 8 7 6 5 4 **3** 2 1 2 3 4 5 6 7 8 9 – B

A – 9 8 7 6 **5** 4 3 2 1 2 3 4 5 6 7 8 9 – C

# M1-9

On quality of life, A and B are equally preferred;  
A is strongly preferred to C; and B is very  
strongly preferred to C.

A – 9 8 7 6 5 4 3 2 **1** 2 3 4 5 6 7 8 9 – B

A – 9 8 7 6 **5** 4 3 2 1 2 3 4 5 6 7 8 9 – C

B – 9 8 **7** 6 5 4 3 2 1 2 3 4 5 6 7 8 9 – C

# M1-9

On the three factors, cost is very strongly preferred to quality of life; cost is moderately preferred to reputation; and reputation is equally to moderately preferred to quality of life.

Cost – 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9 – Quality of life

Cost – 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9 – Reputation

Reputation – 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9 – Quality of life



# M1-9

Develop the pairwise comparison matrices that would be used with the AHP. What university should Gina select?

# Tugas kelompok (2 orang)

Buatlah laporan singkat mengenai keputusan yang melibatkan berapa kriteria dan alternatif dengan menggunakan metode Analytical Hierarchy Process (AHP). Laporan harus mengandung komponen berikut:

1. Deskripsi keputusan yang akan diambil
  - Kriteria-kriteria dan alternatif-alternatif yang terlibat (minimal melibatkan 4 kriteria dan 3 alternatif)
  - Struktur hirarki
2. Pengukuran dan pengumpulan data
  - Pairwise comparison untuk masing-masing kriteria dan alternatif
3. Perhitungan bobot (Normalized weights)
  - Perhitungan bobot untuk masing-masing kriteria dan alternatif
  - Perhitungan consistency ratio (CR), jika diperlukan.
4. Solusi untuk permasalahan
  - Perhitungan skor keseluruhan
  - Keputusan yang direkomendasikan