

Teori Pengambilan Keputusan

Week 11 – Analytical Hierarchy
Process

Analytical Hierarchy Process (AHP)

- AHP is a decision approach designed to aid in the solution of **complex multiple criteria problems** in a number of application domain
- AHP is a decision making tool that decomposes **a complex problem into a multilevel hierarchical structure of objective**, criteria, sub-criteria and alternatives
- AHP is designed to cope with **both the rational and the intuitive** to select the best from a number of alternatives evaluated with respect to several criteria

Analytical Hierarchy Process (AHP)

- In this process, the decision maker carries out **simple pair-wise comparison judgments** (to judge the importance of each criteria) which are then used to develop overall priorities for ranking the alternatives
- AHP allows for inconsistency in the judgments and provides a means to improve consistency

Analytical Hierarchy Process (AHP)

- Principles the AHP:
 - Decomposition: structuring the elements of the problem into a hierarchy
 - Comparative judgments: generating a matrix of pairwise comparisons of all elements in a level with respect to each related element in the level immediately above it where the principal right eigenvector of the matrix provides ratio scaled priority ratings for the set elements compared
 - Synthesis of priorities: calculating the global or composite priority of the elements at the lowest level of the hierarchy

Analytical Hierarchy Process (AHP)

- Rationality in AHP is defined as:
 - Focusing on the goal of solving the problem
 - Knowing enough about a problem to develop a thorough structure of relations and influences
 - Having enough knowledge and experience and access to knowledge and experience of others to access the priority of influence and dominance (importance, preference or likelihood to the goal as appropriate) among the relations in the structure
 - Allowing for difference in opinion with an ability to develop a best compromise

Analytical Hierarchy Process (AHP)

Involving Four Phase :

1. Structuring the decision problem
 - Problem decomposition
 - Define the criteria
 - Design of hierarchy
2. Measuring and collecting data
 - Pair-wise comparison procedure
 - Rate the alternatives

Analytical Hierarchy Process (AHP)

Involving Four Phase :

3. Determining the normalized weights
 - Calculate the weight of the criteria
 - Calculate the consistency ratio (CR)
4. Synthesis-finding solution to the problem
 - Compute the overall score
 - Overall decision

Example AHP (1)

1. Structuring the decision problem

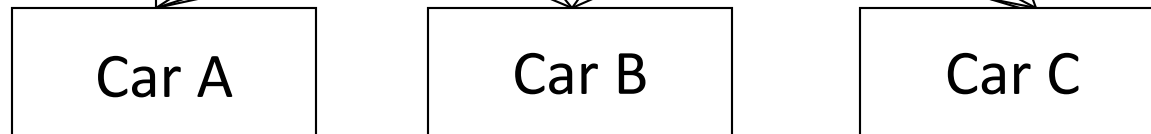
Level 1:
Goal

Buy the best
car

Level 2:
criteria



Level 3:
Alternatives



Example AHP (2)

2. Measuring and collecting data (1)

Pair-wise Comparison Procedure

- Comparisons ask 2 questions:
 - Which is more important with respect to the criterion?
 - How strongly?
- Matrix shows results of all such comparisons
- Typically use a nine point scale (1-9 scale)
- Requires $n(n-1)/2$ judgments
- Information and the priority weights of elements may be obtained from a decision maker using direct questioning or questionnaire method
- Inconsistency may arise
- Relationship between two elements that share a common parent in the hierarchy

Example AHP (3)

2. Measuring and collecting data (2)

We use the following for pairwise comparison:

1. —Equally preferred (sama)
2. —Equally to moderately preferred
3. —Moderately preferred (cukup disukai)
4. —Moderately to strongly preferred
5. —Strongly preferred (lebih disukai)
6. —Strongly to very strongly preferred
7. —Very strongly preferred (sangat lebih disukai)
8. —Very to extremely strongly preferred
9. —Extremely preferred

Example AHP (4)

2. Measuring and collecting data (3)

- Establish priorities:
 - The priorities of the four criteria in terms of over all goal
 - The priorities of the three cars in terms of the price criterion
 - The priorities of the three cars in terms of the MPG criterion
 - The priorities of the three cars in terms of the comfort criterion
 - The priorities of the three cars in terms of the style criterion

Example AHP (5)

2. Measuring and collecting data (4)

Criterion	Price	MPG	Comfort	Style
Price	1	3	2	2
MPG		1		
Comfort		4	1	
Style		4	2	1
Total				

Pair-wise Comparison in terms of all four criteria

Example AHP (5)

2. Measuring and collecting data (4)

Criterion	Price	MPG	Comfort	Style
Price	1	3	2	2
MPG	0.33	1	0.25	0.25
Comfort	0.5	4	1	0.5
Style	0.5	4	2	1
Total	2.33	12	5.25	3.75

Pair-wise Comparison in terms of all four criteria

Example AHP (6)

3. Determining the normalized weights (1)

Price	MPG	Comfort	Style	Priority vector
0.43	0.25	0.38	0.53	0.3982
0.14	0.08	0.05	0.07	0.0851
0.21	0.33	0.19	0.13	0.2179
0.21	0.33	0.38	0.27	0.2988

$= 1 / \text{sum of column 1}$
 $= 1 / (1 + 0.33 + 0.5 + 0.5)$
 $= 0.43$

Priority vector in terms of all four

$= (0.43 + 0.25 + 0.38 + 0.53) / 4$
 $= 0.3982$

Example AHP (7)

Calculate the Consistency Ratio (1)

1. Define the relative priority

Criterion	Price	MPG	Comfort	Style	Priority vector
Price	0.43	0.25	0.38	0.53	0.40
MPG	0.14	0.08	0.05	0.07	0.08
Comfort	0.21	0.33	0.19	0.13	0.22
Style	0.21	0.33	0.38	0.27	0.30

Example AHP (8)

Calculate the Consistency Ratio (2)

2. Calculate the Consistency vector

Vector Priority = $1.69 / 0.40 = 4.23$

Criterion	Price (0.40)	MPG (0.08)	Comfort (0.22)	Style (0.30)
Price	1	3	2	2
MPG	0.33	1	0.25	0.25
Comfort	0.5	4	1	0.5
Style	0.5	4	2	1

$= 1 * 0.4 = 0.4$

$= 4 * 0.08 = 0.34$

Criterion.	Price	MPG	Comfort	Style	Row Sumation	Consistency vector
Price	0.40	0.25	0.44	0.60	1.69	4.23
MPG	0.13	0.08	0.05	0.07	0.35	4.07
Comfort	0.20	0.34	0.22	0.15	0.91	4.16
Style	0.20	0.34	0.44	0.30	1.27	4.26

Example AHP (9)

Calculate the Consistency Ratio (3)

3. Calculate the Consistency Index (CI)

$$\lambda = \frac{\text{Consistency vector}_1 + \text{Consistency vector}_1 + \dots + \text{Consistency vector}_n}{n}$$

$$\lambda = \frac{4.23 + 4.07 + 4.16 + 4.26}{4} = 4.18$$

$$CI = \frac{\lambda - n}{n - 1}$$

$$CI = \frac{4.18 - 4}{4 - 3} = 0.06$$

Example AHP (10)

Calculate the Consistency Ratio (4)

4. Calculate Consistency Ratio (CR)

$$CR = \frac{CI}{RI}$$

- The acceptable CR range varies according to the size of matrix: 0,10
- If CR is equal to or less than that value, it implies that the evaluation within the matrix is acceptable or indicates a good level of consistency in the comparative judgments represented in that matrix. Otherwise, inconsistency has occurred and the evaluation process should therefore be reviewed, reconsidered and improved

Example AHP (11)

Calculate the Consistency Ratio (5)

4. Calculate Consistency Ratio (CR)

$$CR = \frac{CI}{RI}$$

Where:

Size of matrix (n)	RI	Size of matrix (n)	RI
1	0	5	1.12
2	0	6	1.24
3	0.58	7	1.32
4	0.90	8	1.41

Example AHP (12)

Calculate the Consistency Ratio (6)

4. Calculate Consistency Ratio (CR)

$$CR = \frac{CI}{RI}$$

$$CR = \frac{0.06}{0.89} = 0.068 \leq 0.10$$

Therefore, the matrix is consistent

NOTES: Do this calculation for all matrices to check the consistency of the judgment

Example AHP (13)

2. Measuring and collecting data (5)

Price	Car A	Car B	Car C
Car A	1		
Car B	3	1	
Car C	4	2	1

Pair-wise Comparison in terms of Price

Example AHP (15)

2. Measuring and collecting data (6)

MPG	Car A	Car B	Car C
Car A	1		
Car B	4	1	
Car C	6	3	1

Pair-wise Comparison in terms of MPG

Example AHP (17)

2. Measuring and collecting data (7)

	Car A	Car B	Car C
Comfort			
Car A	1	2	8
Car B		1	6
Car C			1

Pair-wise Comparison in terms of Comfort

Example AHP (19)

2. Measuring and collecting data (8)

Style	Car A	Car B	Car C
Car A	1		4
Car B	3	1	7
Car C			1

Pair-wise Comparison in terms of Style

Example AHP (21)

4. Synthesis-finding solution to the problem (1)

Alternative	Price	MPG	Comfort	Style
Car A	0.123	0.087	0.593	0.265
Car B	0.32	0.274	0.341	0.655
Car C	0.557	0.639	0.065	0.08
Overall	0.398	0.085	0.218	0.299

Example AHP (22)

4. Synthesis-finding solution to the problem (2)

- Car A: $0.398 \times 0.123 + 0.085 \times 0.087 +$
 $0.0218 \times 0.593 + 0.299 \times 0.265 = 0.265$
- Car B: $0.398 \times 0.32 + 0.085 \times 0.274 +$
 $0.0218 \times 0.341 + 0.299 \times 0.655 = 0.421$
- Car C: $0.398 \times 0.557 + 0.085 \times 0.639 +$
 $0.0218 \times 0.065 + 0.299 \times 0.08 = 0.314$

Final AHP ranking : B – C – A

EXERCISE

M1-9

Tina adalah mahasiswa Administrasi Bisnis yang akan lulus, dan ia berencana untuk melanjutkan pendidikannya dan mengambil MBA. Tina telah diterima di tiga universitas, dan sekarang ia harus memutuskan universitas mana yang akan dipilihnya. Ia telah memutuskan untuk membandingkan dua universitas dalam satu waktu (pairwise comparison).

M1-9

Dalam hal biaya, Universitas B lebih disukai daripada A; B cukup disukai daripada C; dan C cukup disukai daripada A.

B – 9 8 7 6 (5) 4 3 2 1 2 3 4 5 6 7 8 9 – A

B – 9 8 7 6 5 4 (3) 2 1 2 3 4 5 6 7 8 9 – C

C – 9 8 7 6 5 4 (3) 2 1 2 3 4 5 6 7 8 9 – A

M1-9

Dalam hal reputasi, A sangat lebih disukai daripada B; C cukup disukai daripada B; dan A lebih disukai daripada C.

A – 9 8 **7** 6 5 4 3 2 1 2 3 4 5 6 7 8 9 – B

C – 9 8 7 6 5 4 **3** 2 1 2 3 4 5 6 7 8 9 – B

A – 9 8 7 6 **5** 4 3 2 1 2 3 4 5 6 7 8 9 – C

M1-9

Dalam kualitas hidup, A dan B sama-sama disukai; A lebih disukai daripada C; dan B sangat lebih disukai daripada C.

A – 9 8 7 6 5 4 3 2 **1** 2 3 4 5 6 7 8 9 – B

A – 9 8 7 6 **5** 4 3 2 1 2 3 4 5 6 7 8 9 – C

B – 9 8 **7** 6 5 4 3 2 1 2 3 4 5 6 7 8 9 – C

M1-9

Pada ketiga faktor tersebut, biaya sangat lebih disukai dari kualitas hidup; biaya cukup disukai daripada reputasi; dan reputasi antara sama – cukup disukai daripada kualitas hidup.

Biaya – 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9 – Kualitas Hidup

Biaya – 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9 – reputasi

reputasi – 9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9 – kualitas hidup

M1-9

Buatlah matriks perbandingan yang dapat digunakan untuk prosedur AHP. Universitas manakah yang harus dipilih Tina?

Tugas kelompok (2 orang)

Buatlah laporan singkat mengenai keputusan yang melibatkan berapa kriteria dan alternatif dengan menggunakan metode Analytical Hierarchy Process (AHP). Laporan harus mengandung komponen berikut:

1. Deskripsi keputusan yang akan diambil
 - Kriteria-kriteria dan alternatif-alternatif yang terlibat (minimal melibatkan 4 kriteria dan 3 alternatif)
 - Struktur hirarki
2. Pengukuran dan pengumpulan data
 - Pairwise comparison untuk masing-masing kriteria dan alternatif
3. Perhitungan bobot (Normalized weights)
 - Perhitungan bobot untuk masing-masing kriteria dan alternatif
 - Perhitungan consistency ratio (CR), jika diperlukan.
4. Solusi untuk permasalahan
 - Perhitungan skor keseluruhan
 - Keputusan yang direkomendasikan